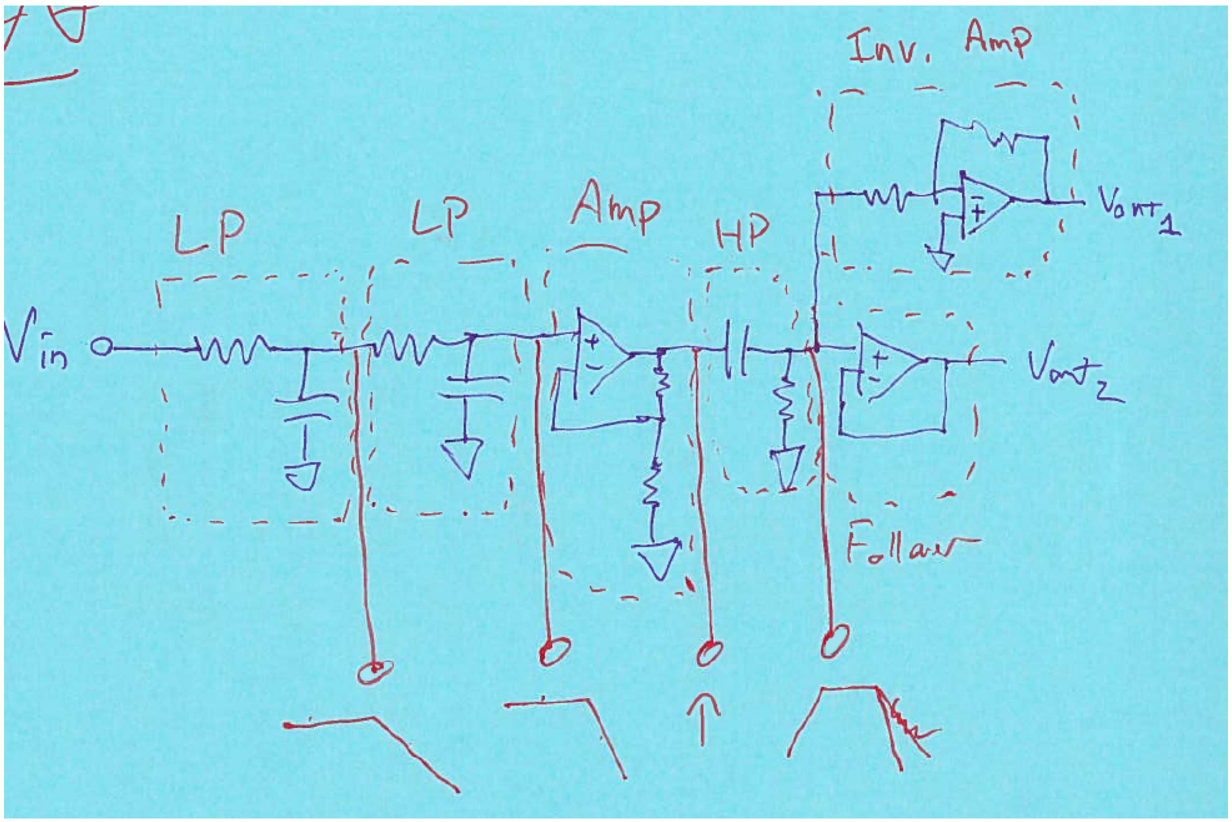
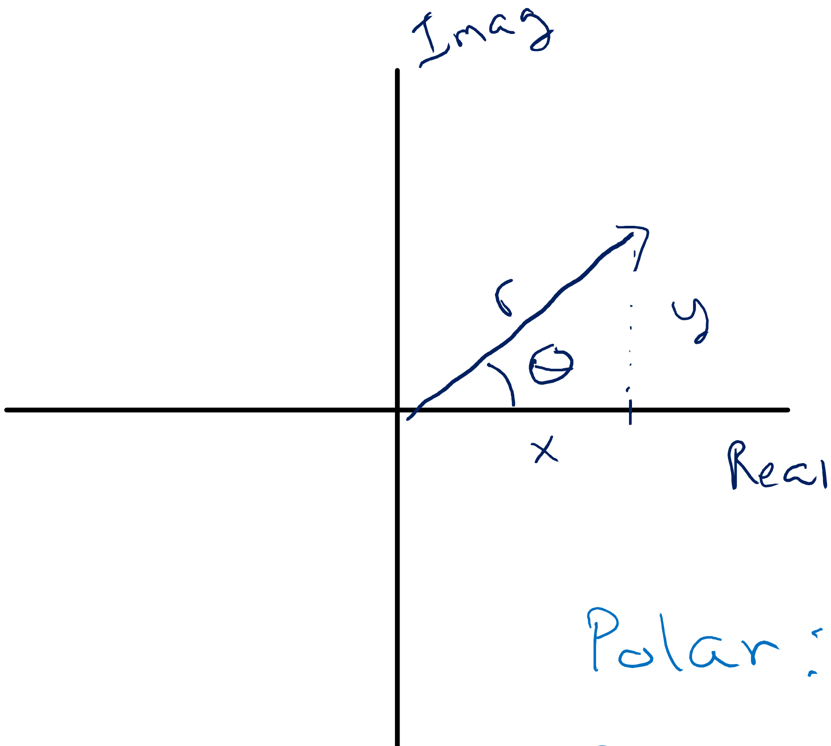

Why can't we be together?
It's complex
√ -1

There’s an easier way…



But first we need to refresh on complex numbers



$z = x + yj$ ← rect

$i = j = \sqrt{-1}$

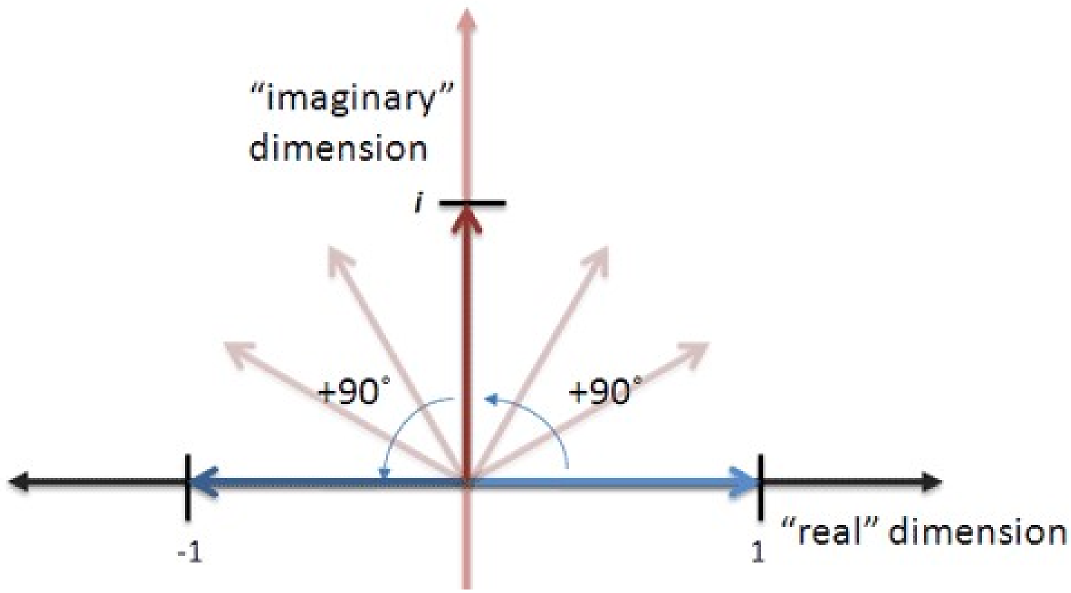
$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}(y/x)$

Polar: $z = r(\cos\theta + j\sin\theta)$

Exp or Amp-Phase: $z = re^{j\theta}$

Rotate 1 to -1



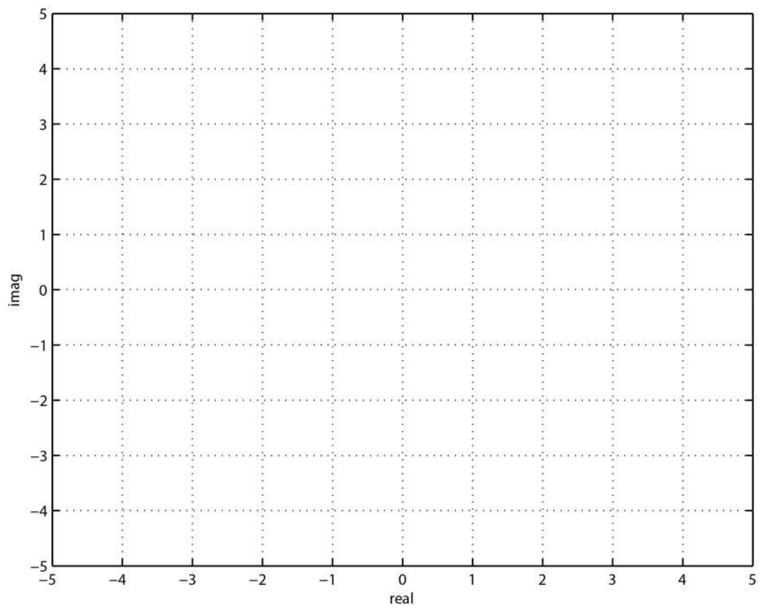
https://betterexplained.com/articles/a-visual-intuitive-guide-to-imaginary-numbers/

Visual explanations of complex numbers

- https://betterexplained.com/articles/a-visual-intuitive-guide-to-imaginary-numbers/
- https://jackschaedler.github.io/circles-sines-signals/sincos.html
- https://jackschaedler.github.io/circles-sines-signals/complex.html
- https://jackschaedler.github.io/circles-sines-signals/euler.html

Worksheet Problems 1-4

You may want a calculator for some ugly numbers and conversions. Look up properties if you need them.



$$z_1 = \sqrt{2}\, e^{j\pi/4} = \sqrt{2}\left(\cos(\pi/4) + j\sin(\pi/4)\right)$$

$$z_2 = 5\, e^{.927j}$$

$$z_1 + z_2 = 4 + 5j$$

$$z_1 - z_2 = -2 - 3j$$

Worksheet Problems 5-6, discuss 7

$$z_1 z_2 = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

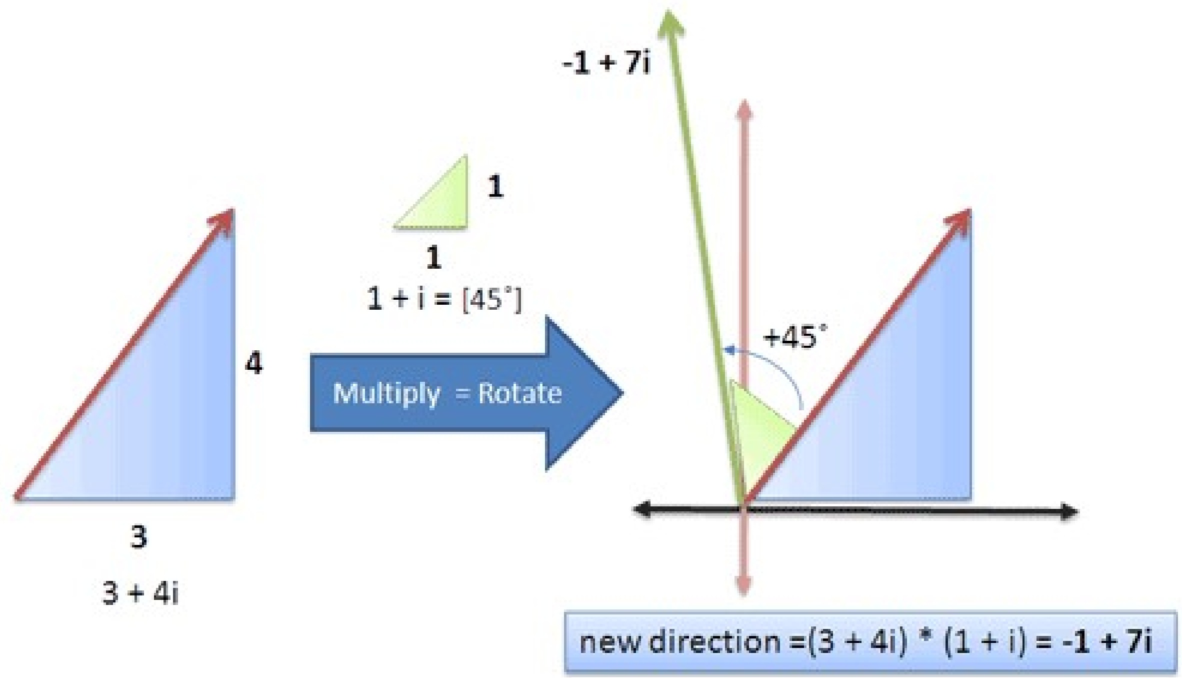
$$z_1 / z_2 = r_1/r_2 \, e^{j(\theta_1 - \theta_2)}$$

$$z_1^4$$

$$z^n = (r e^{j\theta})^n = r^n e^{jn\theta}$$

$$= r^n (\cos n\theta + j \sin n\theta)$$

Applying Complex Numbers



https://betterexplained.com/articles/a-visual-intuitive-guide-to-imaginary-numbers/

De Moivre’s theorem

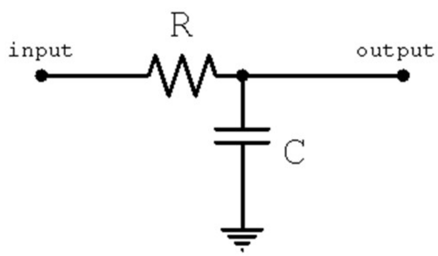
Low-Pass Filters (again!)

$\mathbf{V}$ = bold = complex

$\omega \Rightarrow$ freq (rad/s)

$\theta \Rightarrow$ phase (rad)

$$V_{in}(t) = A\sin(\omega t) \Rightarrow \mathbf{V}_{in}\, e^{j\omega t}$$



$$\mathbf{V} = (x + yj)$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \operatorname{atan}(y/x)$$

$$V_{in}(t) = (x+yj)\,e^{j\omega t}$$

$$= \left(r e^{j\theta}\right) e^{j\omega t}$$

$$= r e^{j(\omega t + \theta)}$$

$$= r\left(\cos(\omega t + \theta) + j\sin(\omega t + \theta)\right)$$

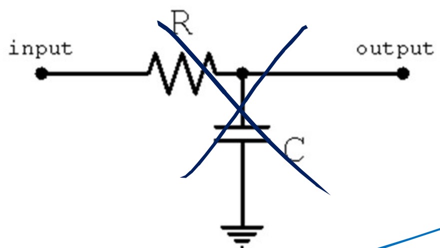
$$z_3 = \frac{1}{1+\omega j} \qquad \left(\frac{1}{1+\omega j}\right)\left(\frac{1-\omega j}{1-\omega j}\right) = \frac{1-\omega j}{1-\omega^2 j^2} = \frac{1-\omega j}{1+\omega^2}$$

$$z_3 = \frac{1}{1+\omega^2} - j\,\frac{\omega}{1+\omega^2}$$

$$r = \sqrt{\left(\frac{1}{1+\omega^2}\right)^2 + \left(\frac{\omega}{1+\omega^2}\right)^2} = \sqrt{\frac{1+\omega^2}{(1+\omega^2)^2}} = \sqrt{\frac{1}{1+\omega^2}}$$

$$\theta = \operatorname{atan}\left(\frac{-\omega/\cancel{1+\omega^2}}{1/\cancel{1+\omega^2}}\right) = \operatorname{atan}(-\omega) = -\operatorname{atan}(\omega)$$

$\rangle$



$$Z_3 = \frac{1}{1+\omega j}$$

$$r = \sqrt{x^2+y^2}$$

$$\boxed{\frac{1}{re^{j\theta}} = \frac{1}{r} e^{-j\theta}}$$

$$x + yj = r e^{j\theta}$$

$$1 + \omega j = \sqrt{1+\omega^2}\, e^{j\,\mathrm{atan}(\omega)}$$

$$\frac{1}{1+\omega j} \qquad \frac{1}{\sqrt{1+\omega^2}\, e^{j\,\mathrm{atan}(\omega)}} = \frac{1}{\sqrt{1+\omega^2}} e^{-j\,\mathrm{atan}(\omega)}$$

Worksheet Part II

- You may want to use Matlab or python for 2 & 3