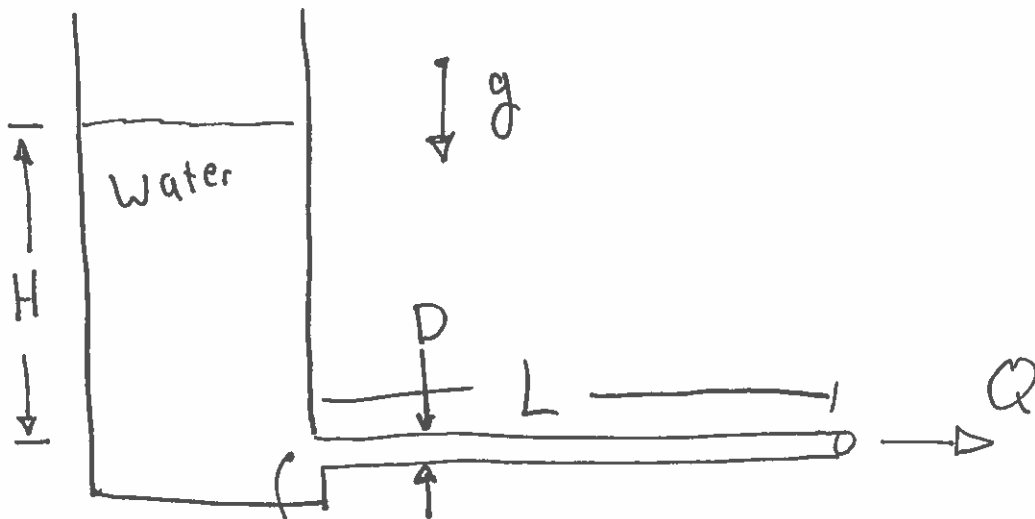


①



Pressure here is $\rho g H$ where ρ is density of water.

Q is volumetric flow rate

Q is measured in m^3/second .

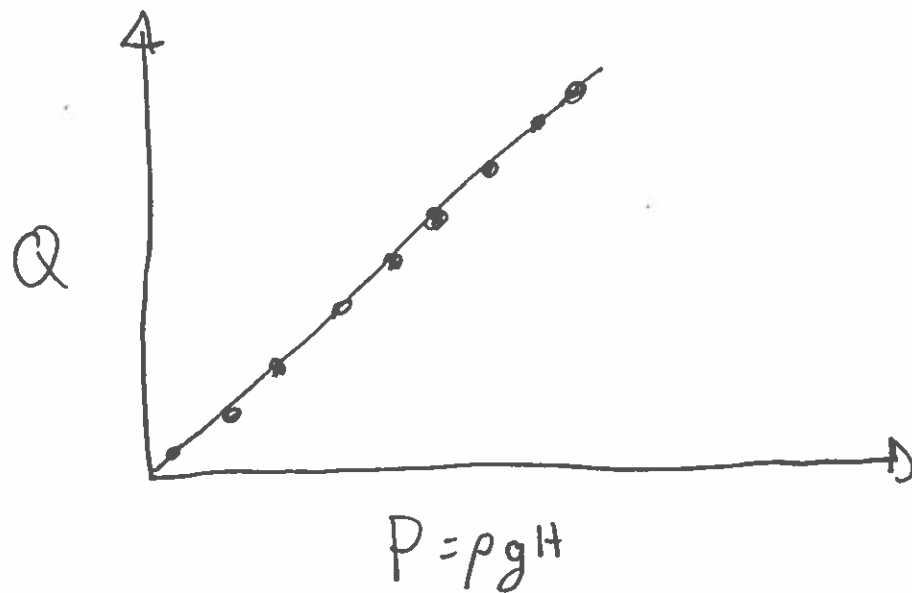
Note that pressure increases as depth, H , increases.

What do we expect?

- Higher pressure (deeper water) \Rightarrow Higher flow
- Increase in pipe length or decrease in diameter will result in lower flow.

(2)

Imagine an experiment where we adjust the height of water in the tank and measure the resulting flow. We can plot the data as:



Imagine data follows straight line.

Define resistance as:

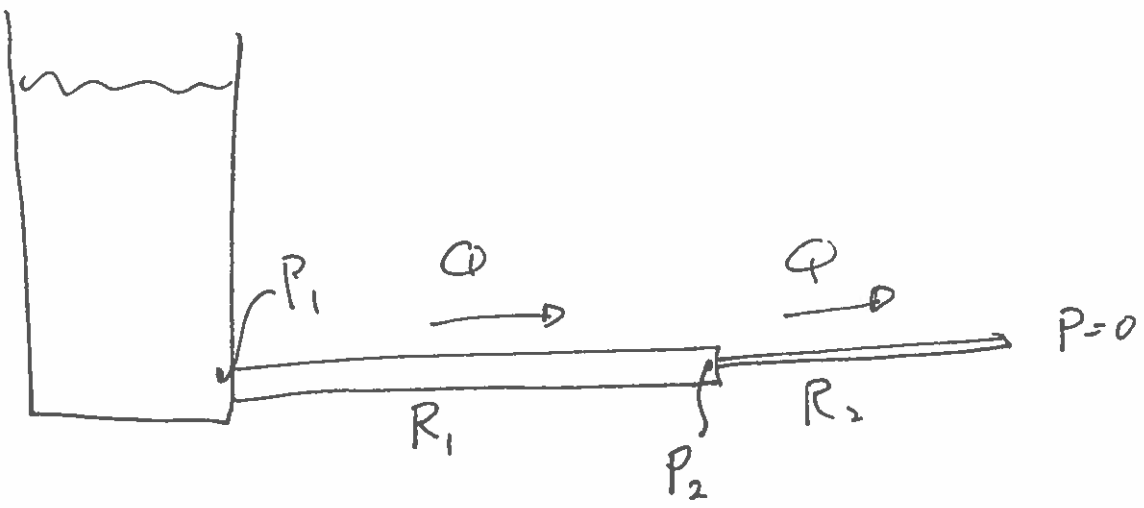
$$R = \frac{P}{Q}$$

OR -

$$\boxed{P = QR}$$

When R is a constant, we have a simple law to predict flow \leftrightarrow Pressure relation.

3



Two pipes in series
Resistance of each is known.

$$\left. \begin{aligned} (P_1 - P_2) &= Q R_1 \\ P_2 &= Q R_2 \end{aligned} \right\} \text{ Holds for each pipe.}$$

Add the equations.

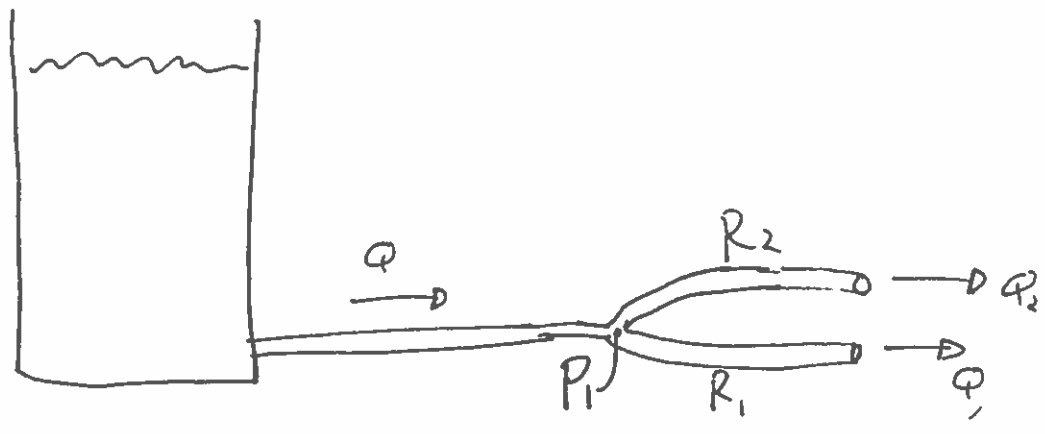
$$P_1 - P_2 + P_2 = Q R_1 + Q R_2$$

$$\boxed{P_1 = Q(R_1 + R_2)}$$

Note that the piping system has a total resistance

of $R_{\text{equivalent}} = R_1 + R_2$

The resistances add up.



Imagine pipe splits. $Q_1 + Q_2 = Q$

Relationship must hold if amount of water is conserved.

Intuitively if $R_1 \gg R_2$ then most of flow goes through pipe #2.

Relations for this system are:

$$P_1 = Q_1 R_1$$

$$P_1 = Q_2 R_2$$

$$Q = Q_1 + Q_2$$

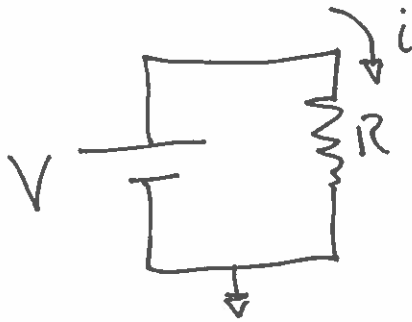
Making Substitutions

$$Q = \frac{P_1}{R_1} + \frac{P_1}{R_2} = P_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

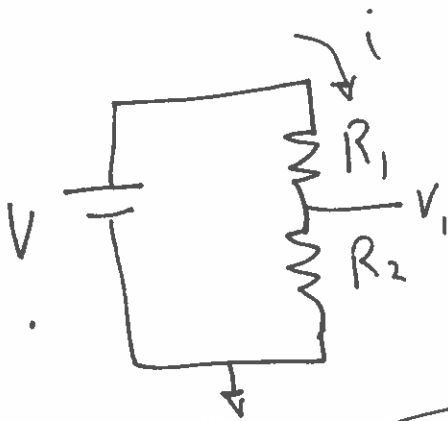
$$P_1 = Q \left(\underbrace{\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}}_{\text{equivalent resistance.}} \right)$$

5

Analogy to electrical circuits



$$V = iR \quad \text{ohms Law}$$



$$V = i(R_1 + R_2) \quad \text{overall}$$

$$V_1 = iR_2 \quad \text{for } R_2$$

combine to get

$$V_1 = V \frac{R_2}{R_1 + R_2}$$

Voltage divider.

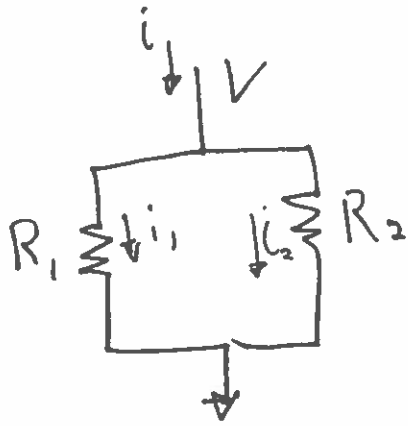
$$\text{If } R_1 \gg R_2 \quad V_1 \Rightarrow 0$$

$$\text{If } R_2 \gg R_1 \quad V_1 \Rightarrow V$$

$$\text{if } R_1 = R_2 \quad V_1 = \frac{V}{2}$$

6

Resistors in parallel



$$i_1 + i_2 = i$$

$$V = i_1 R_1$$

$$V = i_2 R_2$$

Combining gives

$$i = \frac{V}{R_1} + \frac{V}{R_2} = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Rearrange to get

$$V = i \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right) \quad \text{Equivalent Resistor}$$

Can write as

$$V = i \left(\frac{R_1}{1 + R_1/R_2} \right)$$

When $R_2 \gg R_1$ equivalent $R \rightarrow R_1$