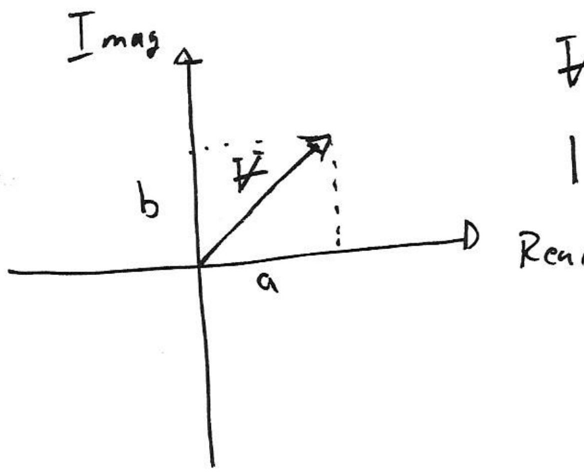
Last Week

- We represent sinusoidal, periodic signals as

$$V(t) = \underbrace{\mathbb{V}}_{\equiv \text{ Complex amplitude}} e^{j\omega t} \qquad \boxed{j = \sqrt{-1}}$$



$$\mathbb{V} = a + bj$$

$$|\mathbb{V}| = \sqrt{a^2 + b^2}$$

$$\theta = \operatorname{atan}\left(b/a\right)$$

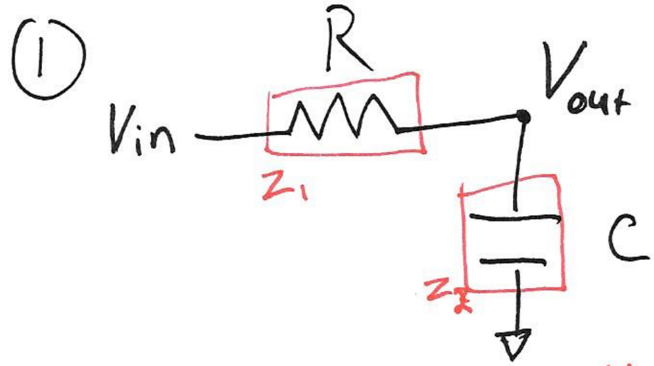
- Complex impedance:

$$Z = \frac{\mathbb{V}}{I} \qquad \text{where } V(t) = \mathbb{V} e^{j\omega t}, \quad i(t) = I e^{j\omega t}$$

$\mathbb{V}$ and I are complex amplitudes of Voltage and Current.

- Resistor $Z = R$
- Capacitor $Z = \dfrac{1}{j\omega C}$

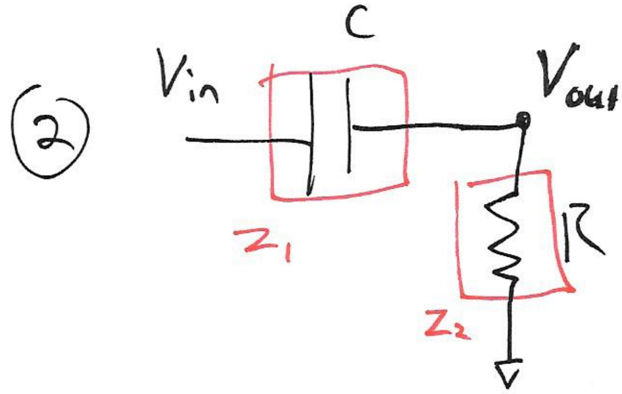
- To analyze w/ impedance,
  - Replace ~~at~~ all R+C w/ Z
  - Analyze as though resistors
  - Substitute for Z at the end.



Voltage Divider

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{R + 1/j\omega C} = \boxed{\frac{1}{1 + j\omega RC}}$$



$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{R}{R + \frac{1}{j\omega C}} = \boxed{\frac{j\omega RC}{1 + j\omega RC}}$$

```
clear;

w = logspace(-3,3,100);
j = sqrt(-1);
RC = 1;


Vout = 1./(1+j*w*RC)
subplot(2,1,1)
loglog(w,abs(Vout))
xlabel('\omega (rad/s)'); ylabel('V_{out}')

subplot(2,1,2)
semilogx(w,angle(Vout)*180/pi)
xlabel('\omega (rad/s)'); ylabel('\theta (deg.)')


figure(1)
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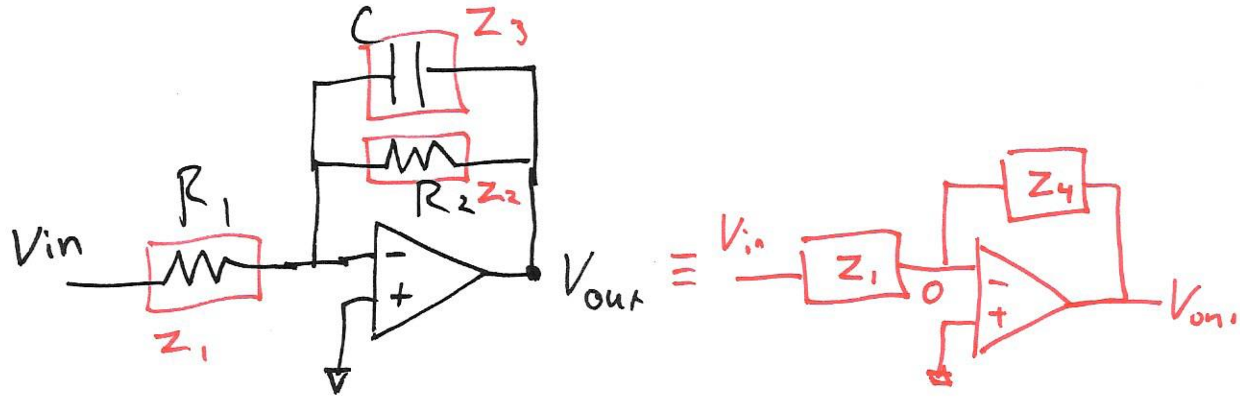
Plot example for #1

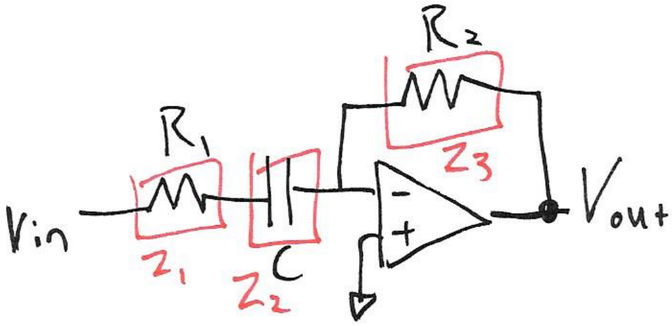
(3)



$$\frac{-V_{out}}{Z_4} = \frac{V_{in}}{Z_1} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{-Z_4}{Z_1} = -\frac{\left(\frac{1}{1/Z_2 + 1/Z_3}\right)}{Z_1}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\frac{1}{1/R_2 + j\omega C}}{R_1} = \boxed{\frac{-R_2/R_1}{1 + j\omega R_2 C}}$$

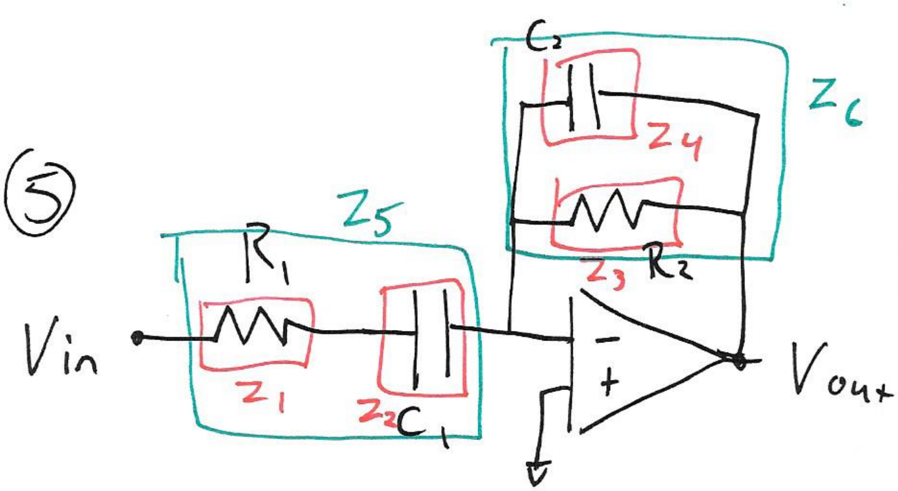
(4)



looking at result from #3 $\frac{V_{out}}{V_{in}} = \frac{-Z_3}{Z_1 + Z_2}$

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + 1/j\omega C} = \frac{R_2 j\omega C}{1 + j\omega C R_1} = \boxed{\frac{(R_2/R_1)\, j\omega C R_1}{1 + j\omega C R_1}}$$

(5)



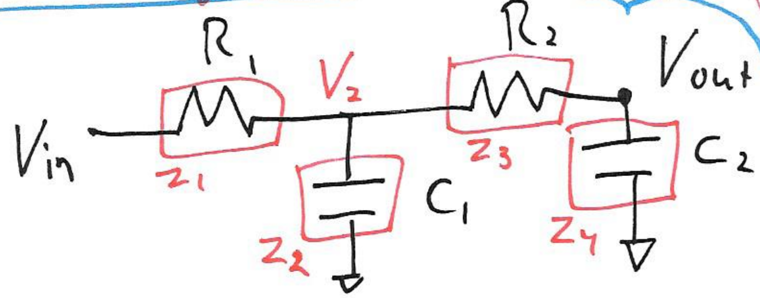
$Z_5 = Z_1 + Z_2$

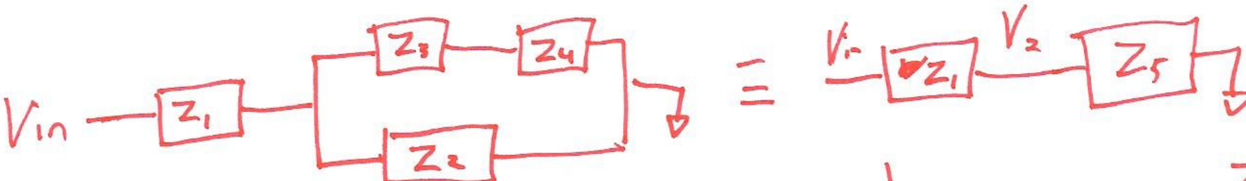
$Z_6 = \dfrac{1}{1/Z_3 + 1/Z_4}$

$$\frac{V_{out}}{V_{in}} = \frac{-Z_6}{Z_5} = \frac{\frac{-1}{1/Z_3 + 1/Z_4}}{Z_1 + Z_2} = \frac{\frac{-1}{1/R_2 + j\omega C_2}}{R_1 + \frac{1}{j\omega C_1}}$$

$$\frac{V_{out}}{V_{in}} = -\left(\frac{j\omega C_1}{1 + j\omega R_1 C_1}\right)\left(\frac{R_2}{1 + j\omega C_2 R_2}\right) = \underbrace{\left(\frac{-j\omega R_1 C_1}{1 + j\omega R_1 C_1}\right)}_{\text{High Pass}} \underbrace{\left(\frac{1}{1 + j\omega R_2 C_2}\right)}_{\text{Low Pass}} \underbrace{\left(\frac{R_2}{R_1}\right)}_{\text{amplify!}}$$

(6)





$$Z_5 = \frac{1}{\frac{1}{Z_2} + \frac{1}{Z_3 + Z_4}} = \frac{Z_2}{1 + \frac{Z_2}{Z_3 + Z_4}}$$

$$\left.\begin{aligned} \frac{V_2}{V_{in}} &= \frac{Z_5}{Z_1 + Z_5} \\ \frac{V_{out}}{V_2} &= \frac{Z_4}{Z_3 + Z_4} \end{aligned}\right\} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{Z_4}{Z_3 + Z_4} \cdot \frac{Z_5}{Z_1 + Z_5} \Rightarrow \text{Next Page}$$

$$Z_5 = \frac{Z_2}{1 + \frac{Z_2}{Z_3+Z_4}} = \frac{\frac{1}{j\omega C_1}}{1 + \frac{\frac{1}{j\omega C_1}}{R_2 + \frac{1}{j\omega C_2}}} = \frac{1}{j\omega C_1 + \frac{1}{R_2 + \frac{1}{j\omega C_2}}}$$

$$= \frac{1}{j\omega C_1 + \frac{j\omega C_2}{1 + j\omega C_2 R_2}} = \frac{1}{j\omega C_1 \left[1 + \frac{C_2/C_1}{1 + j\omega C_2 R_2}\right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_4}{Z_3+Z_4} \cdot \frac{Z_5}{Z_1+Z_5}$$

$$\hookrightarrow \frac{Z_4}{Z_3+Z_4} = \frac{\frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{1}{1 + j\omega C_2 R_2}$$

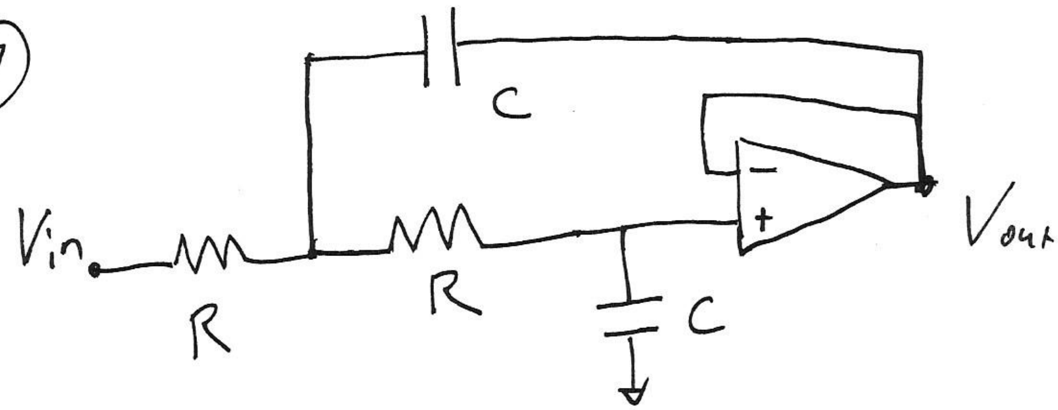
$$\frac{Z_5}{Z_1+Z_5} = \frac{\frac{1}{j\omega C_1} \cdot \frac{1}{1 + \frac{C_2/C_1}{1 + j\omega C_2 R_2}}}{R_1 + \frac{1}{j\omega C_1} \frac{1}{1 + \frac{C_2/C_1}{j\omega C_2 R_2}}} = \frac{1}{j\omega R_1 C_1 \left[1 + \frac{C_2/C_1}{1 + j\omega R_2 C_2}\right] + 1}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega C_2 R_2} \cdot \frac{1}{1 + j\omega C_1 R_1 \left(1 + \frac{C_2/C_1}{1 + j\omega R_2 C_2}\right)}}$$

if $\frac{C_2}{C_1} << 1$

then acts as two low passes!

(7)



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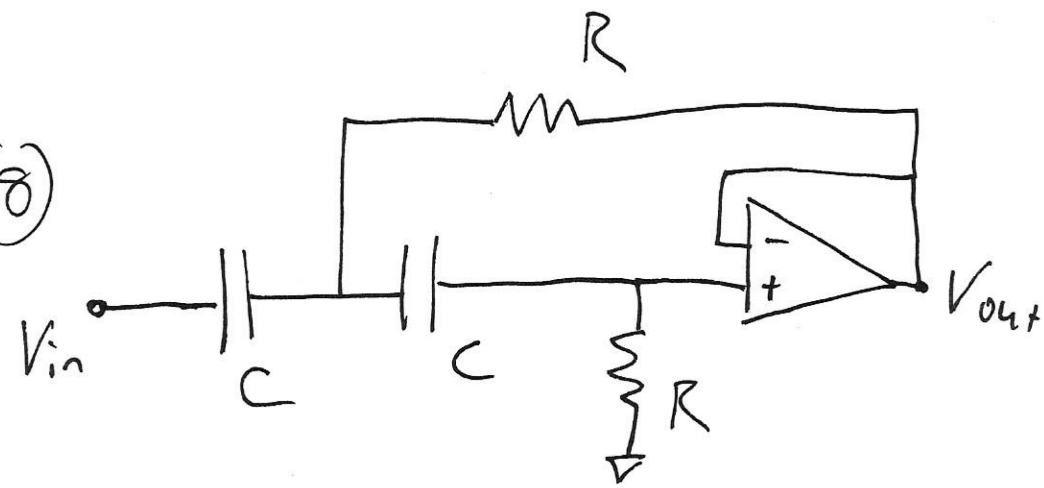

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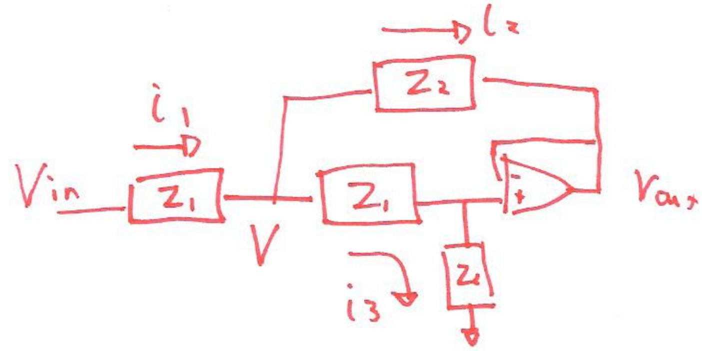

$$i_1 = \frac{V_{in} - V}{Z_1} \qquad i_3 = \frac{V}{Z_1 + Z_2}$$

$$i_2 = \frac{V - V_{out}}{Z_2} \qquad \frac{V_{out}}{V} = \frac{Z_2}{Z_1 + Z_2} \Rightarrow V = V_{out}\frac{(Z_1 + Z_2)}{Z_2}$$

$$i_1 = i_2 + i_3$$

$$\frac{V_{in} - V}{Z_1} = \frac{V - V_{out}}{Z_2} + \frac{V}{Z_1 + Z_2}$$

$$\frac{V_{in}}{Z_1} - \frac{V_{out}}{Z_1}\frac{(Z_1 + Z_2)}{Z_2} = V_0\frac{(Z_1 + Z_2)}{Z_2^2} - \frac{V_0}{Z_2} + \frac{V_{out}\cancel{(Z_1 + Z_2)}}{\cancel{(Z_1 + Z_2)}\, Z_2}$$

$$\frac{V_{in}}{Z_1} = V_{out}\left[\frac{1}{Z_2} + \frac{1}{Z_1} + \frac{1}{Z_2}\left(\frac{Z_1}{Z_2} + 1\right) - \cancel{\frac{1}{Z_2}} + \cancel{\frac{1}{Z_2}}\right]$$

$$V_{in} = V_{out}\left(\frac{Z_1}{Z_2} + 1 + \frac{Z_1}{Z_2}\left(\frac{Z_1 + Z_2}{Z_2}\right)\right]$$

$$V_{in} = V_{out}\left(\left(\frac{Z_1 + Z_2}{Z_2}\right)\left(1 + \frac{Z_1}{Z_2}\right)\right) = V_{out}\left(\frac{Z_1 + Z_2}{Z_2}\right)^2$$

So/

$$\frac{V_{out}}{V_{in}} = \left(\frac{Z_2}{Z_1 + Z_2}\right)^2$$

for #7

$$\frac{V_{out}}{V_{in}} = \left(\frac{1/j\omega C}{R + 1/j\omega C}\right)^2 = \left(\frac{1}{1 + j\omega RC}\right)^2 = (\text{Low Pass})^2$$

for #8

$$\frac{V_{out}}{V_{in}} = \left(\frac{R}{R + 1/j\omega C}\right)^2 = \left(\frac{j\omega RC}{1 + j\omega RC}\right)^2 = (\text{High Pass})^2$$