Introduction to Sensors, Instrumentation, and Measurement

October 29, 2015

1.3 More Stupid Op-Amp Tricks

In this section, we shall consider several additional useful op-amp circuits, using the ideas that we developed in the Section 1.1 to analyze and to reason about their behavior. As before, we shall assume that each circuit is made from a rail-to-rail op amp that is powered from a single-ended supply from $0\,\mathrm{V}$ to V_{dd} unless otherwise stated.

1.3.1 Difference Amplifier

Consider the circuit shown in Fig. 6 comprising one op amp and four resistors. Here, there are two input voltages, V_{in1} and V_{in2}, which couple respectively into the op amp's inverting and noninverting inputs through two matched resistors, each of value R_1. The op amp's output, which is also the output of the circuit as a whole is fed back into the op amp's inverting input through a resistor of value R_2, and a reference voltage, V_{ref}, couples into the op amp's noninverting input through a second resistor of value R_2.

In the bottom half of the circuit, because the op amp's inverting input draws no current, all of the current flowing through R_1 must also flow through R_2. By applying Ohm's law, we have that

$$\underbrace{\frac{V_{\mathrm{in2}} - V_{\mathrm{n}}}{R_1}}_{I_1} = \underbrace{\frac{V_{\mathrm{n}} - V_{\mathrm{out}}}{R_2}}_{I_2},$$

which we can rearrange to find that

$$V_{\mathrm{out}} = -\frac{R_2}{R_1} V_{\mathrm{in2}} + \left(1 + \frac{R_2}{R_1}\right) V_{\mathrm{n}}. \qquad (1)$$

In the top half of the circuit, because the op amp's noninverting input draws no current, we can relate the voltage, $V_{\mathrm{p}} - V_{\mathrm{ref}}$, across R_2 to the total voltage, $V_{\mathrm{in1}} - V_{\mathrm{ref}}$, across both R_1 and R_2 using the voltage divider rule, yielding

$$V_{\mathrm{p}} - V_{\mathrm{ref}} = (V_{\mathrm{in1}} - V_{\mathrm{ref}}) \frac{R_2}{R_1 + R_2},$$

which we can solve, in turn, for V_{p}, obtaining

$$V_{\mathrm{p}} = V_{\mathrm{ref}} + (V_{\mathrm{in1}} - V_{\mathrm{ref}}) \frac{R_2}{R_1 + R_2}. \qquad (2)$$

Now, by the second observation in Section 1.1, in order for V_{out} not to be stuck at one of the rails, we must have that

$$V_{\mathrm{n}} = V_{\mathrm{p}},$$

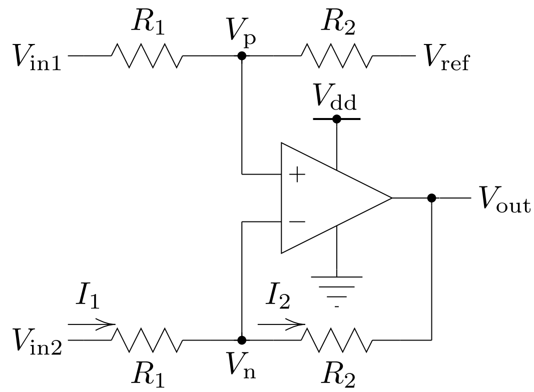

Figure 6: Schematic of an op-amp difference amplifier circuit. Here the op amp's noninverting input voltage, V_p, is set to somewhere between V_{in1} and V_{ref} through a resistive voltage divider. The op amp's output voltage will adjust itself so that the op amp's inverting input voltage, V_n, becomes equal to V_p. Note that, in this circuit, V_n is related to V_{in1} and V_{out} in precisely the same way as V_p is related to V_{in1} and V_{ref}. If the op amp's output is not stuck at one of the rails, the output voltage is given by $V_{out} = V_{ref} + A\,(V_{in1} - V_{in2})$, where $A = R_2/R_1$ is the gain of the circuit.

so we can substitute Eq. 2 into Eq. 1 to find an expression for V_{out}, which is given by

$$\begin{aligned}
V_{out} &= -\frac{R_2}{R_1}V_{in2} + \left(1 + \frac{R_2}{R_1}\right)V_{ref} + \left(1 + \frac{R_2}{R_1}\right)\frac{R_2}{R_1 + R_2}\,(V_{in1} - V_{ref}) \\
&= V_{ref} - \frac{R_2}{R_1}\,(V_{in2} - V_{ref}) + \frac{R_1 + R_2}{R_1}\cdot\frac{R_2}{R_1 + R_2}\,(V_{in1} - V_{ref}) \\
&= V_{ref} - \frac{R_2}{R_1}\,(V_{in2} - V_{ref}) + \frac{R_2}{R_1}\,(V_{in1} - V_{ref}) \\
&= V_{ref} + \underbrace{\frac{R_2}{R_1}}_{A}\,(V_{in1} - V_{in2})\,.
\end{aligned}$$

So, the circuit produces an output voltage that is proportional to the difference between the two inputs by a gain, $A = R_2/R_1$, that is set by the ratio of the two resistors in the circuit. As with the simple inverting amplifier, we can make the gain of this circuit smaller than one, equal to one, or greater than one by our choice of R_1 and R_2.

1.3.2 Three-Op-Amp Instrumentation Amplifier

Consider the circuit shown in Fig. 7 comprising three op amps, six resistors of value R, and one gain resistor of value R_g. This circuit, which is called the *three-op-amp instrumentation amplifier*. We can divide this circuit into two stages. The first stage has two inputs, V_{in1} and V_{in2}, and two outputs, V_3 and V_4, and comprises the two op amps on the left, two resistors of value R and a gain resistor of value R_g. The second stage is the difference amplifier of Fig. 6 with input voltages of V_3 and V_4 and with $R_1 = R_2 = R$, which makes its gain one. Although we could build this circuit from a quad op amp chip and discrete resistors, it is very common

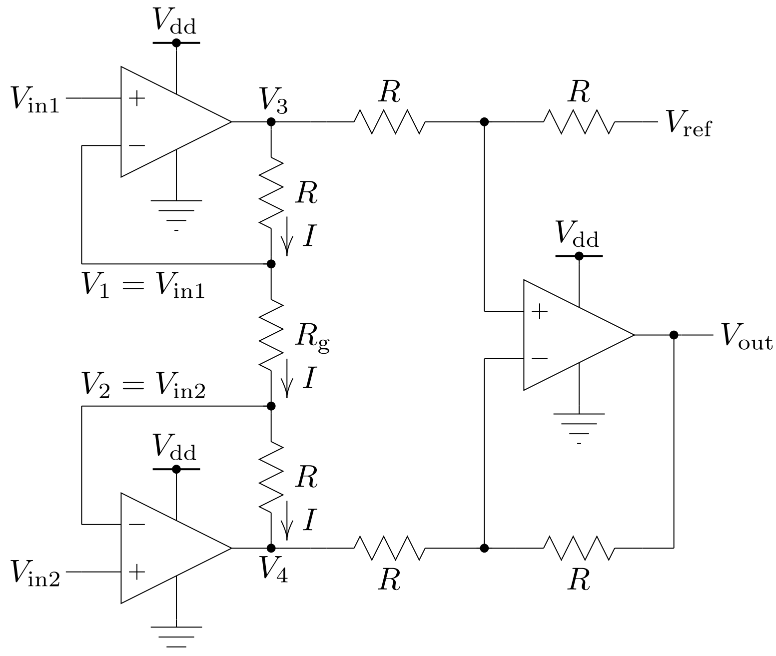

Figure 7: Schematic of a three-op-amp instrumentation amplifier circuit. Note that this circuit effectively has two stages. The first stage has two inputs, V_{in1} and V_{in2}, and two outputs, V_3 and V_4, and comprises the two op amps on the left, two resistors of value R and a gain resistor of value R_g. The second stage is the difference amplifier of Fig. 6 with $R_1 = R_2 = R$, which makes its gain one. If the op amp's outputs are not stuck at the rails, the output voltage is given by $V_{out} = V_{ref} + A\,(V_{in1} - V_{in2})$, where $A = 1 + 2R/R_g$ is the gain of the circuit.

for the three op amps and the six matched resistors to be integrated together on the same chip[1] and an external gain resistor through which we can set the circuit's gain. While it is certainly more convenient to have all those components integrated together on a single chip so we do not have to wire the circuit together, the main advantage of the integrated instrumentation amplifier is that the six resistors of value R can be matched to one another to a much tighter tolerance (e.g., $\pm 0.1\%$ or better) than the $\pm 1\%$ of the discrete resistors that we commonly use in lab. The matching of the four resistors in the second stage is critical to the performance of this circuit.

From our analysis of the difference amplifier in the last section, we know that, assuming that V_{out} is not stuck at one of the rails,

$$V_{out} = V_{ref} + \frac{R}{R}\,(V_3 - V_4) = V_{ref} + (V_3 - V_4)\,. \tag{3}$$

So, to analyze the circuit, all that remains for us to do is to determine how the quantity $V_3 - V_4$ relates to V_{in1} and V_{in2}. Because the op amp's inverting inputs draw no current, by applying Kirchhoff's current law (KCL) to nodes V_1 and V_2, we find that the same current, I,

[1]The AD623 that we have been using in the labs this semester has these components inside it with $R = 50\,\mathrm{k\Omega}$.

must flow through the three vertically oriented resistors, as shown in Fig. 7. Assuming that V_3 is not stuck at one of the rails, V_1 must be equal to V_{in1}. Similarly, if V_4 is not stuck at one of the rails, V_2 must be equal to V_{in2}. Consequently, the voltage across the gain resistor, R_{g}, is just given by the input voltage difference, $V_{\text{in1}} - V_{\text{in2}}$. So, by applying Ohm's law to R_{g}, we have that

$$I = \frac{V_1 - V_2}{R_{\text{g}}} = \frac{V_{\text{in1}} - V_{\text{in2}}}{R_{\text{g}}}. \tag{4}$$

Because the same current, I, flows through all three vertically oriented resistors, they are just connected in series with one another, and so we can consider them to behave as a single equivalent resistor of value $R_{\text{eq}} = R + R_{\text{g}} + R = R_{\text{g}} + 2R$. We know that the total voltage across this equivalent resistor is $V_3 - V_4$, so from Ohm's law, we have that

$$I = \frac{V_3 - V_4}{R_{\text{eq}}} = \frac{V_3 - V_4}{R_{\text{g}} + 2R}. \tag{5}$$

By equating Eq. 4 and Eq. 5 and rearranging, we find that

$$V_3 - V_4 = \frac{R_{\text{g}} + 2R}{R_{\text{g}}} \left(V_{\text{in1}} - V_{\text{in2}}\right) = \left(1 + \frac{2R}{R_{\text{g}}}\right) \left(V_{\text{in1}} - V_{\text{in2}}\right),$$

which we can substitute into Eq. 3 to find that, provided none of the op amp's outputs are stuck at one of the rails,

$$V_{\text{out}} = V_{\text{ref}} + \underbrace{\left(1 + \frac{2R}{R_{\text{g}}}\right)}_{A} \left(V_{\text{in1}} - V_{\text{in2}}\right).$$

From this result, it is apparent both the difference amplifier and the instrumentation amplifier perform the same basic function, so you might be wondering why bother using the instrumentation amplifier? The instrumentation amplifier has two primary advantages over the simpler difference amplifier. First, the instrumentation amplifier's inputs draw no current, because they are op amp inputs, which is not true for the difference amplifier. Second, the gain of the instrumentation amplifier is set by a single component value (i.e., the gain resistor, R_{g}) rather than by a pair of matched components (i.e., R_1 and R_2), as it is with the difference amplifier.

1.3.3 First-Order Low-Pass Amplifier/Filter

Consider the circuit shown in Fig. 8 comprising an op amp, two resistors, R_1 and R_2, and a capacitor C. Here, the op amp's output voltage, V_{out}, which is the output of the circuit as a whole, is fed back into the op amp's inverting input through both R_2 and C, which are in parallel with each other. As with the inverting amplifier circuit, this circuit's input voltage, V_{in}, couples into the op amp's inverting input through R_1. The op amp's noninverting input is held at a constant reference voltage, V_{ref}.

By the second observation of Section 1.1, if V_{out} is not to be stuck at one of the rails, then the op amp's inverting input voltage, V, must be equal to the op amp's noninverting

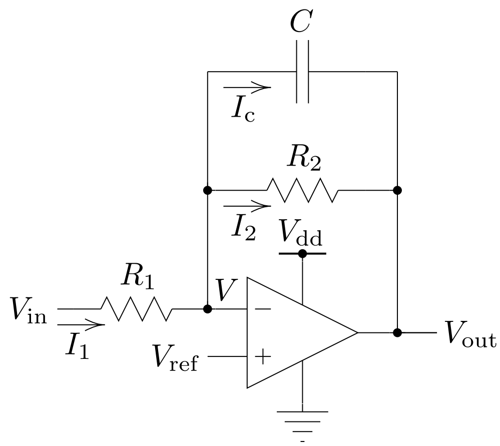

Figure 8: Schematic of a first-order low-pass filter/amplifier circuit. The filter's time constant is given by $\tau = R_2 C$, it's corner frequency is given by $\omega_c = 1/\tau$, and it's pass-band gain is given by $A = -R_2/R_1$.

input voltage, V_{ref}. So, by Ohm's law, we have that

$$I_1 = \frac{V_{in} - V_{ref}}{R_1} \quad \text{and} \quad I_2 = \frac{V_{ref} - V_{out}}{R_2}.$$

Similarly, from the capacitor's current-voltage relationship, we have that

$$I_c = C \frac{d}{dt} \left(V_{ref} - V_{out} \right).$$

Because the op amp's inverting input draws no current, by applying KCL at node V, we have that

$$\underbrace{\frac{V_{in} - V_{ref}}{R_1}}_{I_1} = \underbrace{\frac{V_{ref} - V_{out}}{R_2}}_{I_2} + \underbrace{C \frac{d}{dt} \left(V_{ref} - V_{out} \right)}_{I_c}.$$

By multiplying both sides of this equation by $-R_2$, we have that

$$\underbrace{R_2 C}_{\tau} \frac{d}{dt} \left(V_{out} - V_{ref} \right) + \left(V_{out} - V_{ref} \right) = \underbrace{-\frac{R_2}{R_1}}_{A} \left(V_{in} - V_{ref} \right),$$

where we have defined symbols to represent the filter/amplifier's time constant, $\tau = R_2 C$, and (pass-band) gain, $A = -R_2/R_1$. By defining shifted input and output voltage variables by $U_{in} = V_{in} - V_{ref}$ and $U_{out} = V_{out} - V_{ref}$, we can re-express this equation as

$$\tau \frac{dU_{out}}{dt} + U_{out} = AU_{in},$$

which is identical to the differential equation that governed the simple RC low-pass filter circuit that we analyzed earlier in the course except that the input voltage is amplified by the gain A.

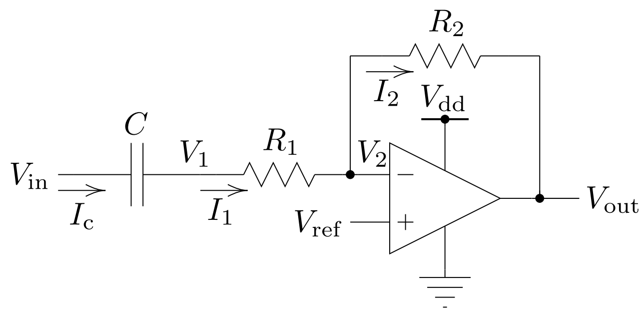

Figure 9: Schematic of a first-order high-pass filter/amplifier circuit. The filter's time constant is given by $\tau = R_1 C$, it's corner frequency is given by $\omega_{\text{c}} = 1/\tau$, and it's pass-band gain is given by $A = -R_2/R_1$.

1.3.4 First-Order High-Pass Amplifier/Filter

Consider the circuit shown in Fig. 9 comprising an op amp, two resistors, R_1 and R_2, and a capacitor C. Here, the op amp's output voltage, V_{out}, which is the output of the circuit as a whole, is fed back into the op amp's inverting input through R_2. The circuit's input voltage, V_{in}, is coupled into the op amp's inverting input through C and R_1, which are connected in series with each other. The op amp's noninverting input is maintained at a constant reference voltage, V_{ref}.

By the second observation of Section 1.1, if V_{out} is not to be stuck at one of the rails, then the op amp's inverting input voltage, V_2, must be equal to the op amp's noninverting input voltage, V_{ref}. So, by Ohm's law, we have that

$$I_1 = \frac{V_1 - V_{\text{ref}}}{R_1} \quad \text{and} \quad I_2 = \frac{V_{\text{ref}} - V_{\text{out}}}{R_2}.$$

Similarly, from the capacitor's current-voltage relationship, we have that

$$I_{\text{c}} = C\frac{d}{dt}\left(V_{\text{in}} - V_1\right). \tag{6}$$

Because the op amp's inverting input draws no current, by applying KCL at node V_2, we have that

$$\underbrace{\frac{V_1 - V_{\text{ref}}}{R_1}}_{I_1} = \underbrace{\frac{V_{\text{ref}} - V_{\text{out}}}{R_2}}_{I_2},$$

which we can rearrange to solve for V_1, thereby obtaining

$$V_1 = V_{\text{ref}} + \frac{R_1}{R_2}\left(V_{\text{ref}} - V_{\text{out}}\right). \tag{7}$$

By substituting Eq. 7 into Eq. 6, we can express I_{c} as

$$\begin{aligned} I_{\text{c}} &= C\frac{d}{dt}\left(V_{\text{in}} - V_{\text{ref}} - \frac{R_1}{R_2}\left(V_{\text{ref}} - V_{\text{out}}\right)\right) \\ &= C\frac{d}{dt}\left(V_{\text{in}} - V_{\text{ref}}\right) + C\frac{R_1}{R_2} \cdot \frac{d}{dt}\left(V_{\text{out}} - V_{\text{ref}}\right). \end{aligned}$$

By applying KCL at node V_1, we have that $I_c = I_1$, but we know that $I_1 = I_2$, so we have that

$$\underbrace{C\frac{d}{dt}\left(V_{\text{in}} - V_{\text{ref}}\right) + C\frac{R_1}{R_2} \cdot \frac{d}{dt}\left(V_{\text{out}} - V_{\text{ref}}\right)}_{I_c} = \underbrace{\frac{V_{\text{ref}} - V_{\text{out}}}{R_2}}_{I_2},$$

which we can rearrange to find that

$$C\frac{R_1}{R_2} \cdot \frac{d}{dt}\left(V_{\text{out}} - V_{\text{ref}}\right) + \frac{V_{\text{out}} - V_{\text{ref}}}{R_2} = -C\frac{d}{dt}\left(V_{\text{in}} - V_{\text{ref}}\right).$$

By multiplying both sides of this equation by R_2, we have that

$$R_1C\frac{d}{dt}\left(V_{\text{out}} - V_{\text{ref}}\right) + \left(V_{\text{out}} - V_{\text{ref}}\right) = -R_2C\frac{d}{dt}\left(V_{\text{in}} - V_{\text{ref}}\right).$$

By multiplying the right-hand side of this equation by unity in the form of R_1/R_1, we have that

$$\underbrace{R_1C}_{\tau}\frac{d}{dt}\left(V_{\text{out}} - V_{\text{ref}}\right) + \left(V_{\text{out}} - V_{\text{ref}}\right) = \underbrace{-\frac{R_2}{R_1}}_{A} \cdot \underbrace{R_1C}_{\tau}\frac{d}{dt}\left(V_{\text{in}} - V_{\text{ref}}\right),$$

where we have defined symbols to represent the filter/amplifier's time constant, $\tau = R_1C$, and (pass-band) gain, $A = -R_2/R_1$. By defining shifted input and output voltage variables by $U_{\text{in}} = V_{\text{in}} - V_{\text{ref}}$ and $U_{\text{out}} = V_{\text{out}} - V_{\text{ref}}$ and by moving the gain, A, inside the time derivative, we can re-express this equation as

$$\tau\frac{dU_{\text{out}}}{dt} + U_{\text{out}} = \tau\frac{d}{dt}\left(AU_{\text{in}}\right),$$

which is identical to the differential equation that governed the simple RC high-pass filter circuit that we analyzed earlier in the course except that the input voltage is amplified by the gain A.