Introduction to Sensors, Instrumentation, and Measurement

October 21, 2015

1 Operational Amplifiers

In this handout, we shall describe the operation and application of the *operational amplifier* (op amp), which is arguably the most versatile analog circuit module that anyone has devised. The word *operational* in the name comes from the days of analog computers in which they were used in conjunction with resistors, capacitors, and other kinds of components, to make circuits that performed mathematical *operations* on time-varying voltage signals, such as multiplication by a fixed constant, addition, subtraction, integration with respect to time, or various combinations of these. Although we would seldom[1] build electronic analog computers these days to solve differential equations or to simulate physical systems, such operations are still quite useful in processing the time-varying signals[2] that arise from sensors. We shall make use of several of these circuits in the labs this semester.

1.1 Op-Amp Operation

Fig. 1 shows the symbol that we typically use in circuit schematics for an op amp. It has two inputs, the *noninverting* or *positive* input (denoted in the symbol by a plus sign) and the *inverting* or *negative* input (denoted in the symbol by a minus sign), and a single output. A change in the noninverting input voltage, V_{pos}, will cause the op amp to change its output voltage, V_{out}, in the same direction (whence the name *noninverting*), whereas a change in the inverting input voltage, V_{neg}, will cause the op amp to change V_{out} in the opposite direction (whence the name *inverting*). In an effort to simplify circuit schematics, we usually use the symbol shown in Fig. 1a, which does not show the op amp's connections to the power supplies. When we need to show these explicitly, we use the symbol shown in Fig. 1b. Here, V_{dd} denotes the positive power supply voltage and V_{ss} denotes the negative power supply voltage. Even when they are not shown explicitly, the power supply voltages are important to keep in mind when working with op amps. Aside from the fact that op amps need power in order to do anything at all, the power supply voltages typically bound both the allowable range of input voltages and the range of output voltages. If one or both of these voltage ranges extends from one power supply voltage to the other, we say that the op amp has a *rail-to-rail* input/output voltage range.

In this course, we will typically use a *single-ended* power supply configuration in which V_{ss} is $0\,\text{V}$ (i.e., ground) and V_{dd} is some positive value (e.g., $+5\,\text{V}$). Another common power supply configuration is a *split* supply in which V_{ss} and V_{dd} are equally spaced around ground (e.g., $\pm 5\,\text{V}$ or $\pm 12\,\text{V}$). Most op amps are specified to function properly over a range of total power supply voltages (i.e., $V_{\text{dd}} - V_{\text{ss}}$), but it is important to read a given op amp's datasheet

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[1] Never say "never." For example, see G. E. R. Cowan, R. C. Melville, and Y. P. Tsividis, "A VLSI Analog Computer/Digital Computer Accelerator," *IEEE Journal of Solid-State Circuits*, vol. 41, no. 1, pp. 42–53, 2006.

[2] For example, a class of filter circuits called *state-variable filters* are precisely analog-computer realizations of the differential equations that describe the behavior of such filters.

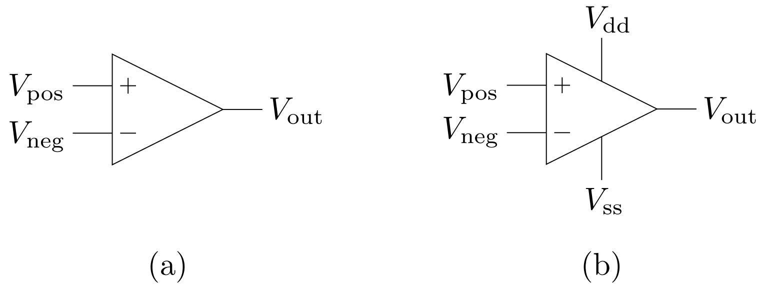

Figure 1: Circuit schematic symbols for an op amp with the power supply connections (a) suppressed and (b) shown explicitly. Here, $V_{\rm pos}$ and $V_{\rm neg}$ are the *noninverting* and *inverting* inputs to the op amp, respectively, and $V_{\rm out}$ is the output of the op amp. In part b, $V_{\rm dd}$ and $V_{\rm ss}$ are the op amp's positive and negative power supply voltages, respectively. Note that the power supply connections are always necessary and important even when they are not shown explicitly in a circuit schematic. The power supply voltages typically bound both the allowable range of the input voltages and the range of the output voltage.

to see if it will function with a particular supply arrangement. For example, this semester we will be using the LMC6484, which is a quad[3] op amp that is specified to work with with total power supply voltages from 3 V up to a maximum of 18 V. So, this op amp will work well on a single-ended supply of +5 V, but if we tried to power it from a split supply of ±12 V, we would burn up the chip.[4] The LMC6484 has a rail-to-rail voltage range on both its inputs and its output. On the other hand, if we had a TL084,[5] which is specified to work with power supply voltages from 10 V up to a maximum of 36 V, it would work fine on a split supply of ±12 V, but it would not function properly on a single-ended supply of +5 V. The input-voltage range of the TL084 typically goes from 3 V above the negative supply voltage to the positive supply voltage. The output-voltage range typically stops about 1.5 V short of both power supply voltages. Neither its input-voltage range nor its output-voltage range extends from rail to rail.[6] Apparently, not all op amps are created equal!

The first thing to know about the operation of op amps is that, by design, their input terminals draw very little current—these input currents, which are typically nanoamps (i.e., $1\,{\rm nA} = 10^{-9}\,{\rm A}$) or less, are sufficiently small compared to the currents flowing in the other circuit elements connected to the op amp's inputs that we can safely neglect them in comparison to these other currents, which are typically on the order of microamps (i.e., $1\,\mu{\rm A} = 10^{-6}\,{\rm A}$) or milliamps (i.e., $1\,{\rm mA} = 10^{-3}\,{\rm A}$). For example, the LMC6484 datasheet indicates that, at room temperature (i.e., 25°C) typical input current values are 20 fA (i.e., $20 \times 10^{-15}\,{\rm A}$) and that the maximum input currents are 4 pA (i.e., $4 \times 10^{-12}\,{\rm A}$). Now, these are *very* small currents. In order to get a sense for them, let's think about the solderless

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[3] A *quad* op amp has four independent op amps on a single chip that all share common power-supply pins. Similarly, A *dual* op amp has two independent op amps on a single chip that share common power-supply pins.

[4] The chip will literally start smoking and would probably get hot enough to melt your breadboard and to burn your finger if you were to touch it.

[5] Another quad op amp that you might find around campus.

[6] When you are working with a total supply voltage of 24 V, who cares about 3 V off the ends? The story is a little different if you are working with at total supply voltage of only 3.3 V!

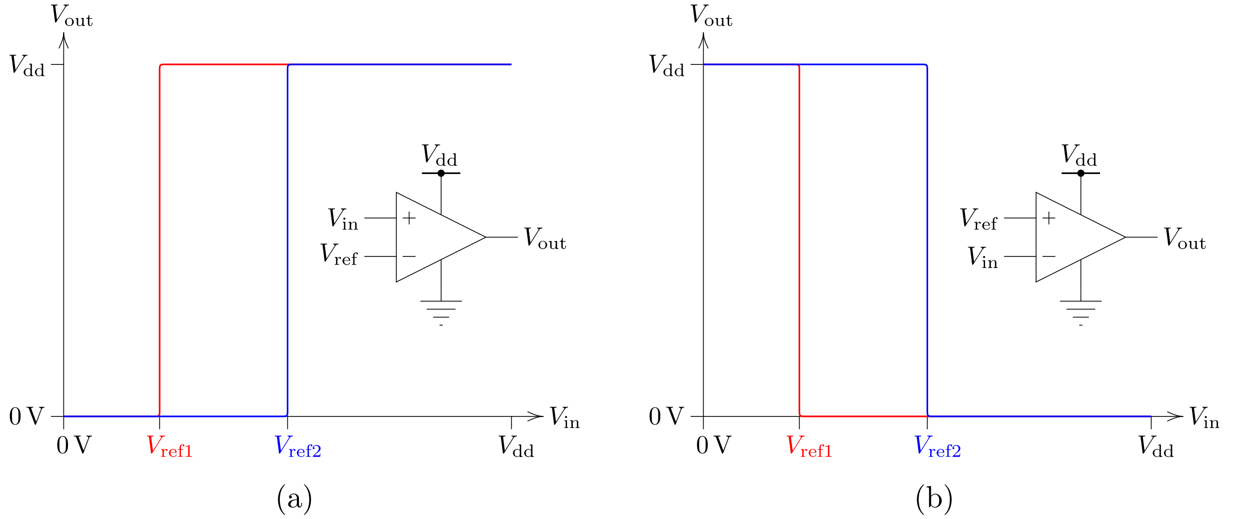

Figure 2: Voltage transfer characteristics (VTCs) of an idealized op amp with a rail-to-rail output-voltage range sweeping (a) the noninverting input and (b) the inverting input for two different values, V_{ref1} and V_{ref2}, of the other input. In each case, note that the place where V_{out} is not saturated at one rail or the other is when the two input voltages are equal (i.e., $V_{\text{in}} = V_{\text{ref}}$).

breadboards that you are using to build circuits in lab this semester. The plastic insulation used in solderless breadboards is typically only guaranteed to exceed 10 GΩ (i.e., $10 \times 10^9\,\Omega$). If we had a one-volt difference between two adjacent rows in a breadboard and the resistance of the insulation were only 10 GΩ, then a *leakage current* of about $1\,\text{V}/10\,\text{G}\Omega = 100\,\text{pA}$ would flow through the plastic insulation—that is 25 times larger than the maximum specified input currents of the LMC6484 and 5,000 times larger than the typical values specified! Similarly, the TL084 datasheet indicates that, at room temperature, the input currents are typically 30 pA but no more than 200 pA—these are certainly not as small as the values specified for the LMC6484, but they are still on the same order as the leakage currents through the breadboard insulation. If we are not going to worry about leakage currents in the breadboards, we certainly should not be concerned about an op amp's input currents!

The second thing to know about op-amp operation is that, if an op amp's input voltages are the same (i.e., $V_{\text{pos}} = V_{\text{neg}}$), V_{out} remains constant. If $V_{\text{pos}} > V_{\text{neg}}$, V_{out} rapidly (i.e., typically in a few microseconds) increases until it *saturates* at the upper limit of its range (e.g., V_{dd} for a rail-to-rail op amp). Similarly, if $V_{\text{pos}} < V_{\text{neg}}$, V_{out} will rapidly decrease until it saturates at the lower limit of its range (e.g., V_{ss} for a rail-to-rail op amp). With these facts in mind, let us consider the *voltage transfer characteristic* (VTC)[7] of an op amp with a rail-to-rail output range operating on a single-ended supply for each of its inputs while the other is held at some constant value, V_{ref}. Fig. 2a shows the VTC for the noninverting input with the inverting input held fixed at V_{ref}. When $V_{\text{in}} < V_{\text{ref}}$, the op amp's output will be saturated at the negative rail (i.e., 0 V). When $V_{\text{in}} > V_{\text{ref}}$, V_{out} will be saturated at V_{dd}. What about when $V_{\text{in}} = V_{\text{ref}}$? Here, *any* value of V_{out} between the rails is compatible with

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[7] A VTC is a (static) plot showing the output voltage of a circuit as a function of its input voltage.

this input condition (i.e., V_{out} would just stay where it happened to be when V_{in} became equal to V_{ref}, which suggests that the VTC is a essentially a vertical line when $V_{\text{in}} = V_{\text{ref}}$. In reality, the VTC will not quite be vertical around $V_{\text{in}} = V_{\text{ref}}$, but the slope, which is called the op amp's *low-frequency gain*, denoted by A_0, can easily be on the order of 10^5 or 10^6, which is *very* steep.[8] Fig. 2b shows the VTC for the inverting input with the noninverting input held fixed at V_{ref}. Here, when $V_{\text{in}} < V_{\text{ref}}$, the op amp has a *positive* input difference, which will drive V_{out} to saturates at V_{dd}. When $V_{\text{in}} > V_{\text{ref}}$, the op amp has a negative input difference, which will drive V_{out} to saturate at the negative rail. As with the VTC of the noninverting input, when $V_{\text{in}} = V_{\text{ref}}$, V_{out} could assume any value between the rails, so the (idealized) VTC would be a vertical line at that point. The import of these VTCs is as follows. If an op amp's output is stuck at the positive rail, we know that $V_{\text{pos}} > V_{\text{neg}}$. On the other hand, if V_{out} is stuck at the negative rail, we know that $V_{\text{neg}} < V_{\text{pos}}$. In order for V_{out} not to be stuck at one of the rails, we must have that $V_{\text{pos}} = V_{\text{neg}}$. For a large, finite low-frequency gain, A_0, we might say more generally that, in order for V_{out} not to be stuck at one of the rails, we must have that

$$|V_{\text{pos}} - V_{\text{neg}}| < \frac{V_{\text{dd}} - V_{\text{ss}}}{A_0}.$$

We will quickly be able both to analyze most of the op-amp circuits that we will be using this semester by using making use of the following two observations:

1. an op amp's input terminals draw negligible current (i.e., $I_{\text{pos}} = I_{\text{neg}} \approx 0\,\text{A}$) and
2. in order for an op amp's output not to be saturated at one of the rails, its input voltages must be nearly equal (i.e., $V_{\text{pos}} \approx V_{\text{neg}}$).

In the next section, we shall illustrate their use by analyzing several common op-amp circuits. However, in some cases that we shall see, which typically involve op-amp circuits that process signals that change rapidly (e.g., ultrasound), we will need to account for the internal dynamics of the op amps. In such cases, we shall assume that the op amp's output voltage changes at a rate that, until it reaches an upper limit called the op amp's *slew rate*, is given by

$$\frac{dV_{\text{out}}}{dt} = \omega_1 \left(V_{\text{pos}} - V_{\text{neg}}\right),$$

where ω_1 is called the op amp's *gain-bandwidth product* or its *unity-gain crossover frequency*, which is an angular frequency expressed in radians per second.[9] The gain-bandwidth product of most op amps is a large number,[10] usually on the order of 10 Mrad/s or more. Consequently, in response to even a slight difference between its input voltages, an op amp will change its output voltage very quickly so long as the input difference persists or until the output saturates at the upper or lower limit of its range. This simple dynamic model is a very good description of real op-amp behavior over a wide range of frequencies, typically

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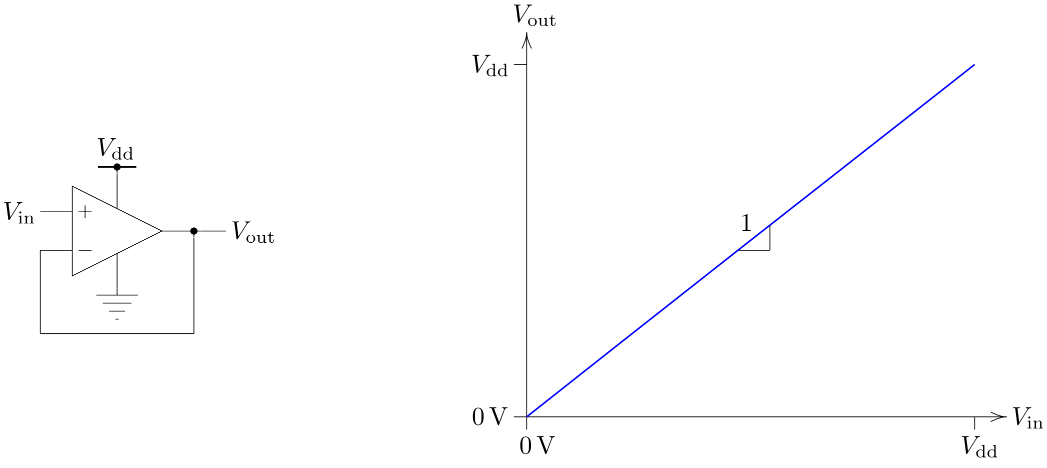
[8] It is also pretty much impossible to measure directly.

[9] Op-amp datasheets typically quote the gain-bandwidth product in hertz (i.e., cycles per second) rather than in radians per second, so recall that $\omega_1 = 2\pi f_1$.

[10] For reference, the typical LMC6484 has a gain-bandwidth product of 9.4 Mrad/s and a slew rate of 1.3 V/μs, and the typical TL084 has a gain-bandwidth product of 18.8 Mrad/s and a slew rate of 13 V/μs.



(a) (b)

Figure 3: Unity-gain follower (a) circuit schematic and (b) VTC. The circuit's input voltage is connected directly to the op amp's noninverting input and op amp's output voltage, which is also the circuit's output voltage, is fed back to the op amp's inverting input. The op amp adjusts the output voltage so that it becomes equal to the input voltage—V_{out} follows V_{in}, whence the circuit's name. The follower's VTC is the identity line, $V_{\text{out}} = V_{\text{in}}$.

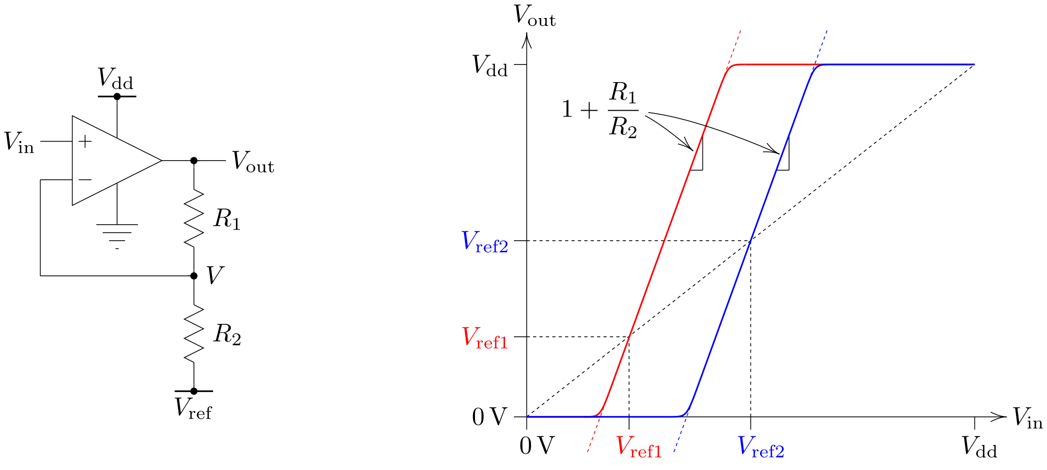
ranging from a few cycles per second to a bit beyond the op amp's unity-gain crossover frequency.

1.2 Stupid Op-Amp Tricks

In this section, we shall consider several several very practical op-amp circuits, using the ideas that we developed in the last section to analyze and to reason about their behavior. Unless otherwise stated, we shall assume that each circuit is made from a rail-to-rail op amp that is powered from a single-ended supply from 0 V to V_{dd}.

1.2.1 Unity-Gain Follower/Buffer

Consider the circuit shown in Fig. 3a, which is called a *unity-gain follower* or sometimes a *buffer*. Here, the input voltage is connected directly to the op amp's noninverting input and the op amp's output voltage, which is also the circuit's output voltage, is fed back into the op amp's inverting input. Suppose that we have fixed V_{in} somewhere between the rails. If V_{out} happened to be less than V_{in}, the op amp's input voltage difference would be positive, and it would respond by increasing V_{out}, reducing the difference between V_{out} and V_{in}. Eventually, V_{out} would become equal to V_{in} and the op amp's output would stay put. Similarly, if V_{out} had started out higher than V_{in}, the op amp's input voltage difference would be negative, and it would respond by decreasing V_{out}, again reducing the difference between V_{out} and V_{in}. Eventually, V_{out} would become equal to V_{in} and the op amp's output would stay put. If we were to change V_{in} from this point, the op amp would adjust V_{out} to become equal to V_{in}. In essence, V_{out} follows V_{in}, whence the name *follower*.



(a) (b)

Figure 4: Noninverting amplifier (a) circuit schematic and (b) VTC for two values of V_{ref}. The circuit's input voltage is connected directly to the op amp's noninverting input and the op amp's output voltage, which is also the circuit's output voltage, is fed back to the op amp's inverting input through a resistive voltage divider to a reference voltage, V_{ref}. The op amp adjusts V_{out} so that its inverting input voltage, V, becomes equal to V_{in}. Because, the change in V is only a fraction (i.e., the divider ratio) of the change in V_{out}, the change in V_{out} to compensate for any given change in V_{in} must be *greater* than the change in V_{in} (whence *amplifier*). Moreover, V changes in the same direction as V_{out}, so V_{out} changes in the same direction as V_{in} (whence *noninverting*). Until V_{out} saturates at the rails, the noninverting amplifier's VTC is given by $(V_{\text{out}} - V_{\text{ref}}) = A\,(V_{\text{in}} - V_{\text{ref}})$, where the gain is $A = 1 + R_1/R_2$, which is the reciprocal of the voltage divider ratio.

Using the second observation of the last section, we could reason that, in order for the circuit's output voltage to not be stuck at one of the rails, $V_{\text{pos}} = V_{\text{neg}}$, but $V_{\text{pos}} = V_{\text{in}}$ and $V_{\text{neg}} = V_{\text{out}}$, so it follows that

$$V_{\text{out}} = V_{\text{in}},$$

so the circuit's VTC is just the identity line, as shown in Fig. 3b. At this point, you might be wondering why anyone would use an op amp connected in this way when a simple wire seems like it would do just as well. The answer comes when we recall the first observation of the last section: an op amp's inputs draw a negligible amount of current. In many cases, drawing current from a node whose voltage we would like to observe might significantly change the value of that voltage. This circuit allows us to observe a voltage somewhere in a circuit, effectively by making a copy of it, without drawing any current from the point we are observing; the op amp will supply any current required by whatever we connect to the output.

1.2.2 Noninverting Amplifier

Consider the circuit shown in Fig. 4a, which is called a *noninverting amplifier*. Here, the input voltage is connected directly to the op amp's noninverting input and the op amp's output voltage, which is also the circuit's output voltage, is fed back to the op amp's inverting input through a resistive voltage divider to a reference voltage, V_{ref}. Suppose that we have fixed V_{in} somewhere between the rails. Because of the voltage divider comprising R_1 and R_2, the op amp's inverting input voltage, V, will lie strictly between V_{out} and V_{ref}, and when V_{out} changes, V will change in the same direction by an amount given by the change in V_{out} times the voltage divider ratio, $R_2/(R_1 + R_2)$. Now, if V happened to be below V_{in}, then the op amp would have a positive input voltage difference and, as a result, would increase its output voltage. The change in V_{out} would give rise to a correlated change in V through the voltage divider, thereby reducing the input difference. This situation would continue until V becomes equal to V_{in}, reducing the input difference to zero, at which point the op amp's output would stop changing. Similarly, if V had started out higher than V_{in}, the op amp would have a negative input voltage difference and, as a result, would decrease V_{out}. Again, the change in V_{out} would give rise to a correlated change in V, thereby reducing the input difference. As before, this situation would continue until the input difference is driven to zero at which point V_{out} would stop changing. If we were to change V_{in}, the op amp would adjust V_{out} so that V becomes equal to the new value of V_{in}. Because the change in V is smaller than the change in V_{out} by the voltage divider ratio, to compensate for a given change in V_{in}, the op amp must change V_{out} in the same direction by an amount that is given by the change in V_{in} times the reciprocal of the divider ratio, whence the name *noninverting amplifier*.

To analyze the circuit, we can make use of the two observations from the last section as follows. By the second observation, in order for V_{out} not to be stuck at one of the rails, we must have that

$$V_{\text{in}} = V_{\text{pos}} = V_{\text{neg}} = V. \tag{1}$$

Moreover, because the op amp's inverting input draws no current from the circuit node between R_1 and R_2, the voltage across R_2, $V - V_{\text{ref}}$, is related to the total voltage across the R_1 and R_2, $V_{\text{out}} - V_{\text{ref}}$, by the voltage divider rule, given by

$$V - V_{\text{ref}} = (V_{\text{out}} - V_{\text{ref}})\,\frac{R_2}{R_1 + R_2},$$

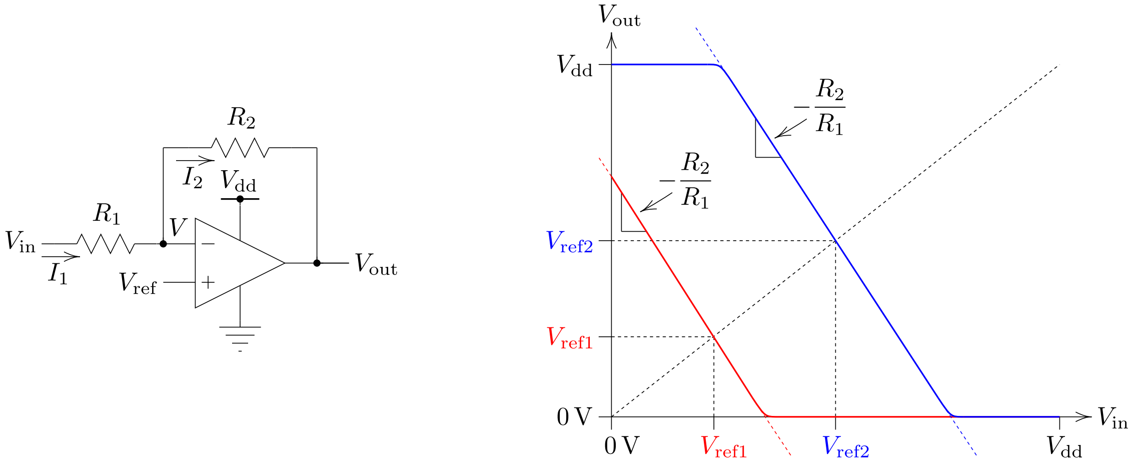
which implies that

$$V_{\text{out}} - V_{\text{ref}} = \left(1 + \frac{R_1}{R_2}\right)(V - V_{\text{ref}})\,.$$

By substituting Eq. 1 into this equation, we obtain an equation for the noninverting amplifier's VTC, which is given by

$$V_{\text{out}} - V_{\text{ref}} = \underbrace{\left(1 + \frac{R_1}{R_2}\right)}_{A}(V_{\text{in}} - V_{\text{ref}})\,,$$

where $A = 1 + R_1/R_2$ is called the *gain* of the noninverting amplifier. Fig. 4b shows a plot of this VTC for two values of V_{ref}. Note that the op amp's output will saturate at both power supply rails whereas this equation, which we obtained by assuming that V_{out} was not stuck against one of the rails, happily keeps on going oblivious to the op amp's bounded output range.



(a) (b)

Figure 5: Inverting amplifier (a) circuit schematic and (b) VTC for two values of V_{ref}. The circuit's input voltage couples to the op amp's inverting input through a resistor, R_1, and the op amp's output, which is also the circuit's output voltage, is fed back to the op amp's inverting input through another resistor, R_2. The op amp's noninverting input is held at a reference voltage, V_{ref}. The op amp adjusts V_{out} so that V becomes or remains equal to V_{ref}. A change in V_{in} has a tendency to change V in the same direction. To compensate, the op amp changes its output in the opposite direction so that V remains constant at or returns to V_{ref} (whence *inverting*). The amount by which V_{out} must change to compensate a given change in V_{in} is set by the ratio of the resistors—the circuit can either amplify, invert (i.e., have a gain of -1), or attenuate, depending on the size of R_2 compared to that of R_1. Until V_{out} saturates at the rails, the inverting amplifier's VTC is given by $(V_{\mathrm{out}} - V_{\mathrm{ref}}) = A\,(V_{\mathrm{in}} - V_{\mathrm{ref}})$, where the gain is $A = -R_2/R_1$.

1.2.3 Inverting Amplifier

Consider the circuit shown in Fig. 5a, which is called an *inverting amplifier*. Here, the input voltage is coupled to the op amp's inverting input through resistor R_1 and the op amp's output voltage, which is also the circuit's output voltage, is fed back to the op amp's inverting input through a resistor R_2. The op amp's noninverting input is connected to a reference voltage, V_{ref}. Suppose that we have fixed V_{in} somewhere between the rails. If V happened to be less than V_{ref}, the op amp would have a positive input voltage difference and, as a result, would increase V_{out}. An increase in V_{out}, in turn, will result in an increase in V, thereby reducing the op amp's input voltage difference. This situation would continue until V becomes equal to V_{ref} at which time V_{out} would stop changing. Similarly, if V were above V_{ref}, the op amp would have a negative input voltage difference and, as a result, would decrease V_{out}. This decrease in V_{out}, in turn, results in a decrease in V. Again, this situation would persist until V becomes equal to V_{ref} at which point V_{out} would stop moving. From this point, an increase V_{in} would result in an increase in V and the op amp having a negative input voltage difference. As before, the op amp would respond by decreasing V_{out}, restoring the equality between V and V_{ref}. If the increase in V_{in} is slow enough compared to the op amp's response time (as characterized by its gain-bandwidth product and slew rate), V will essentially remain fixed and V_{out} will move in the opposite direction from V_{in} to compensate.

To analyze the inverting amplifier, we can make use of our two observations of the last section as follows. By the second observation, in order for V_{out} not to be stuck at one of the rails, we must have that

$$V_{\text{ref}} = V_{\text{pos}} = V_{\text{neg}} = V. \tag{2}$$

By the first observation, the op amp's inverting input draws no current, so all of the current flowing through R_1 must also flow through R_2. So, by applying Ohm's law, we have that

$$\underbrace{\frac{V_{\text{in}} - V}{R_1}}_{I_1} = \underbrace{\frac{V - V_{\text{out}}}{R_2}}_{I_2},$$

which we can rearrange to obtain

$$V_{\text{out}} - V = -\frac{R_2}{R_1}\left(V_{\text{in}} - V\right).$$

By substituting Eq. 2 into this equation, we obtain an equation for the inverting amplifier's VTC, which is given by

$$V_{\text{out}} - V_{\text{ref}} = \underbrace{-\frac{R_2}{R_1}}_{A}\left(V_{\text{in}} - V_{\text{ref}}\right),$$

where $A = -R_2/R_1$ is the gain of the inverting amplifier. Fig. 5b shows a plot of this VTC for two values of V_{ref}. Note that, as was the case with the noninverting amplifier, the op amp's output will saturate at both power supply rails, deviating from this equation, which we derived by assuming that V_{out} was not stuck against one of the rails. Also note that, unlike the noninverting amplifier, we can choose R_1 and R_2 so that the circuit amplifies, inverts (i.e., multiplies by -1), or attenuates.