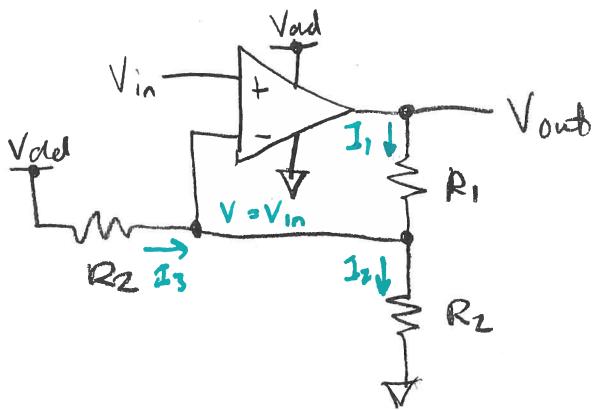


(1)



$$0V < V_{out} < V_{dd} \Rightarrow V \approx V_{in}$$

$$\text{KCL at } V \Rightarrow I_1 + I_3 = I_2$$

$$\Rightarrow \underbrace{\frac{V_{out} - V_{in}}{R_1}}_{I_1} + \underbrace{\frac{V_{dd} - V_{in}}{R_2}}_{I_3} = \underbrace{\frac{V_{in} - 0V}{R_2}}_{I_2}$$

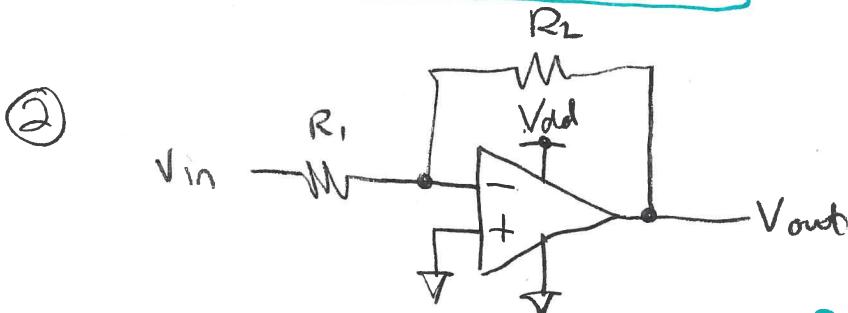
$$(=) \times R_1 \Rightarrow \boxed{V_{out} = \left(1 + \frac{2R_1}{R_2}\right)V_{in} - \frac{R_1}{R_2}V_{dd}}$$

$$(=) - \frac{V_{dd}}{2} \Rightarrow$$

$$V_{out} - \frac{V_{dd}}{2} = \left(1 + 2\frac{R_1}{R_2}\right)V_{in} - \left(\frac{1}{2} + \frac{R_1}{R_2}\right)V_{dd}$$

$$\Rightarrow V_{out} - \frac{V_{dd}}{2} = \left(1 + 2\frac{R_1}{R_2}\right)V_{in} - \left(1 + 2\frac{R_1}{R_2}\right)\frac{V_{dd}}{2}$$

$$\Rightarrow V_{out} - \frac{V_{dd}}{2} = \underbrace{\left(1 + 2\frac{R_1}{R_2}\right)}_A \underbrace{\left(V_{in} - \frac{V_{dd}}{2}\right)}_{U_{in}}$$

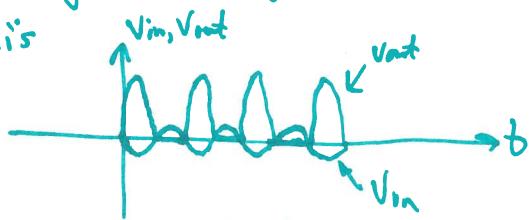


Inverting amplifier, Gain = $-\frac{R_2}{R_1}$, $V_{ref} = 0V$.

Expect $V_{out} = -\frac{R_2}{R_1}V_{in}$. However, V_{out} can't go below 0V!

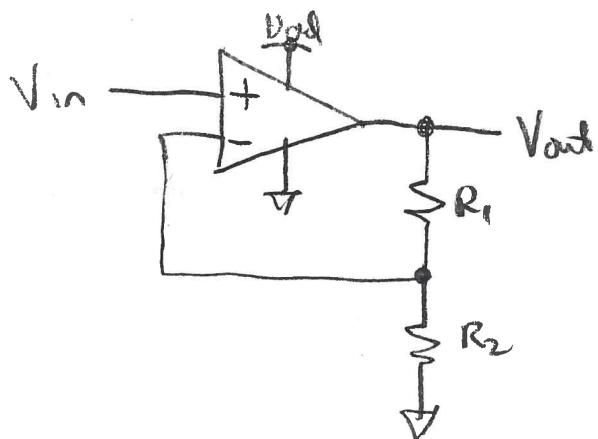
If $V_{in} > 0V$, V_{out} wants to go below 0V, but gets stuck at 0V.

So, if V_{in} can go below ground, the circuit's response will look like this



If the input range were limited to positive voltages, the output will always be stuck at 0V!

(3)

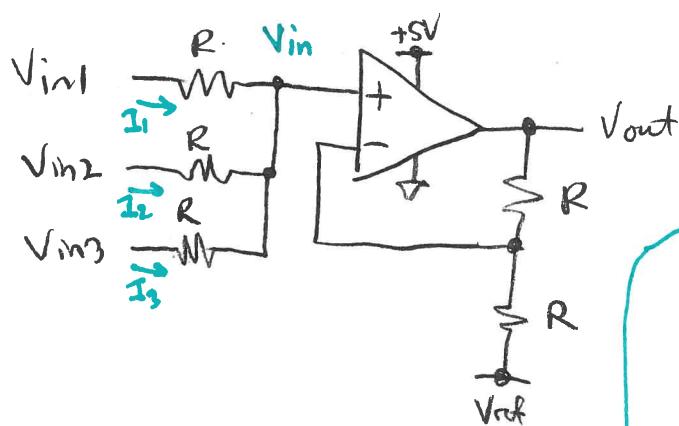
Noninverting amp with $V_{ref} = 0V$

$$V_{out} - 0V = \left(1 + \frac{R_1}{R_2}\right)(V_{in} - 0V)$$

$$\Rightarrow V_{out} = \left(1 + \frac{R_1}{R_2}\right)V_{in}$$

until V_{out} saturates @ V_{dd}

(4)

Noninverting amp w/ $R_1 = R_2 = R$.

$$\Rightarrow A = 2.$$

$$\Rightarrow V_{out} - V_{ref} = 2(V_{in} - V_{ref})$$

$$\Rightarrow V_{out} - V_{ref} = 2\left(\frac{V_{in1} + V_{in2} + V_{in3}}{3} - V_{ref}\right)$$

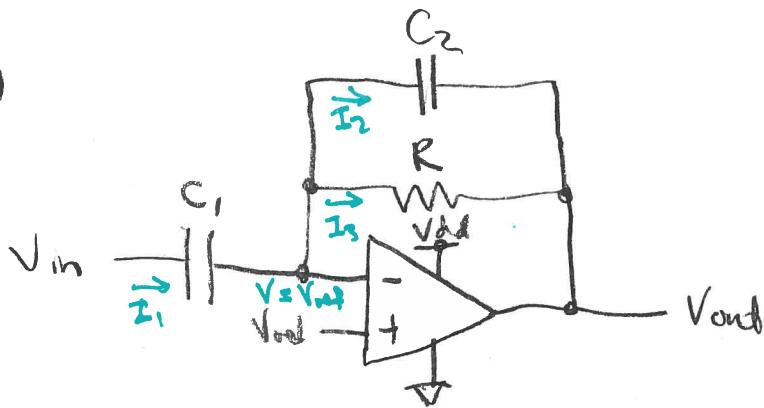
$$KCL @ V_{in} \Rightarrow I_1 + I_2 + I_3 = 0A$$

$$\Rightarrow \underbrace{\frac{V_{in1} - V_{in}}{R}}_{I_1} + \underbrace{\frac{V_{in2} - V_{in}}{R}}_{I_2} + \underbrace{\frac{V_{in3} - V_{in}}{R}}_{I_3} = 0A.$$

$$(=) \times R \Rightarrow V_{in1} + V_{in2} + V_{in3} = 3V_{in}$$

$$\Rightarrow \boxed{V_{in} = \frac{1}{3}(V_{in1} + V_{in2} + V_{in3})}$$

(5)



$$0V < V_{out} < V_{ref} \Rightarrow V = V_{ref}$$

$$I_1 = C_1 \frac{d}{dt} (V_{in} - V_{ref}),$$

$$I_2 = C_2 \frac{d}{dt} (V_{ref} - V_{out}),$$

$$\text{and } I_3 = \frac{V_{ref} - V_{out}}{R}$$

$$\text{KCL at } V \Rightarrow I_1 = I_2 + I_3$$

$$\Rightarrow \underbrace{C_1 \frac{d}{dt} (V_{in} - V_{ref})}_{I_1} = \underbrace{C_2 \frac{d}{dt} (V_{ref} - V_{out})}_{I_2} + \underbrace{\frac{V_{ref} - V_{out}}{R}}_{I_3}$$

$$(=) \times -R \Rightarrow -RC_1 \frac{d}{dt} (V_{in} - V_{ref}) = RC_2 \frac{d}{dt} (V_{out} - V_{ref}) + (V_{out} - V_{ref})$$

$$\Rightarrow \underbrace{RC_2 \frac{d}{dt} (V_{out} - V_{ref})}_{\tau \frac{dU_{out}}{dt}} + (V_{out} - V_{ref}) = - \underbrace{\frac{C_1}{C_2} \cdot RC_2 \frac{d}{dt} (V_{in} - V_{ref})}_{A \tau \frac{dU_{in}}{dt}} \underbrace{(V_{in} - V_{ref})}_{U_{in}}$$

$$\boxed{\tau \frac{dU_{out}}{dt} + U_{out} = A \tau \frac{dU_{in}}{dt}}$$

First-order high-pass filter!

$$\text{when } \tau = RC_2$$

$$A = -\frac{C_1}{C_2}$$