Time response of RC circuit

Consider a resistor and capacitor arranged in series as shown in Figure 1. The voltage at the left side of the resistor will be set by some external source and will be the input voltage, V_{in}. The voltage between the resistor and capacitor will be considered the output of the circuit, V_{out}. Initially $V_{\text{out}} = V_{\text{in}} = 1$. At $t > 0$ the input voltage is instantaneously pulled down to $V_{\text{in}} = 0$.

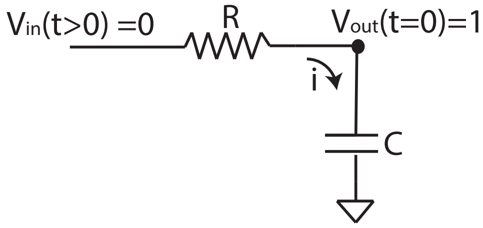

FIG. 1 Circuit that we will analyze. Initially the voltages are set to 1 and then suddenly the input voltage is pulled down to zero.

The current through the resistor at $t > 0$ is

$$i = \frac{V_{\text{in}} - V_{\text{out}}}{R} = \frac{-V_{\text{out}}}{R},$$

and the current through the capacitor is,

$$i = C\frac{dV_{\text{out}}}{dt}.$$

Since the parts are in series, the current through the resistor must equal that through the capacitor,

$$RC\frac{dV_{\text{out}}}{dt} = -V_{\text{out}}.$$

We can notice from the units here, that the product RC has units of time.

Separating the variables of this equation,

$$\frac{dV_{\text{out}}}{V_{\text{out}}} = \frac{-dt}{RC},$$

and integrating yields,

$$\ln(V_{\text{out}}) = \frac{-t}{RC} + B,$$

where B is a constant of integration. Taking the exponential of both sides of the equation and applying the initial condition $V_{\text{out}}(t = 0) = 1$ yields,

$$V_{\text{out}} = \mathrm{e}^{-t/RC}.$$

The result here is the same as in the hydraulic analogy.